ECE: Special Topics in Systems and Control

Optimal Control: Theory and Applications

Course #: 18879C/96840SV
Semester: Fall 2012
Breadth Area: Artificial Intelligence, Robotics and Control

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Lecture Information:
Times: Tues/Thurs 10AM-11:20AM (PST); Fri 1-2:20PM (PST)
Location: SV: Rm. 118; **PGH HH 1107**
Lectures will be available online via adobe connect:
http://cmusv.adobeconnect.com/optimalcontrol/

Teaching Assistant: TBD
Email: TBD
Office Hours: TBD
Location: SV: Rm. 118; **PGH TBD**

Course Description:

This course will cover the fundamentals optimal control theory and include applications from current research in aeronautics. Specific topics include: extrema of functions and functionals with and without constraints, Lagrange multipliers, calculus of variations, variational approach to optimal control with and without constraints, interior points, bang-bang control, Pontryagin maximum principle, Dynamic programming and the Hamilton-Jacobi-Bellman equations, singular optimal control, and stochastic optimal control.

Prerequisites: This course is intended for advanced undergraduate and beginning graduate students. The prerequisites are ordinary differential equations and 18-470 – Fundamentals of Control. It is helpful, but not required, to have taken or to take concurrently: 18-771 – Linear Systems which is currently offered by Professor Sinopoli without conflict. While the course will utilize elements of real and functional analysis, prior exposure to these topics it is not required. However, students with such previous exposure will manifestly find the material more digestible. At the conclusion of the course, stochastic control may be covered and the essential elements from probability theory will be developed as needed.

Course Content Overview

1. Introduction to Optimal Control (2 Lectures)
   - Applications in aeronautics: trajectory optimization and wing shaping control
   - Historical perspectives: Calculus of Variations and Optimal Control
     - References: McShane [12] and Goldstine [13]
   - Further introduction and notions of calculus of variations; Brachistochrone problem; numerical example
     - References: Gelfand [4], Pinch [2], and Gibbons[10]
   - Optimal Control Problem Statement; 5 examples; geometric solution of the rocket railroad problem
     - Reference: Evans [1], chapter 1.
2. Review of some Calculus Facts and Optimization in Finite Dimensional Spaces (2 Lectures)
   - Fundamental theorem of calculus; mean-value theorem; Taylor's theorem; chain rule; integration by parts; Leibnitz's differentiation rule
   - Definition of global and local maxima, minima in one and several variables; conditions for critical points – necessary and sufficient conditions; critical, end, and discontinuous points
     - References: Pinch [2]
   - Sufficiency conditions for optimality; feasible directions and global minima; compactness and Weirerstrass theorem (prove later)
     - Reference: Liberzon [11]
   - Extremum problems with constraints; Lagrange Multipliers involving one and several variables.
     - Reference: Amazigo [5])
   - Examples and HW problems
3. Some Elements of Functional Analysis (1 Lecture)
   - Metric Spaces; open sets, closed sets, neighborhoods, convergence, Cauchy sequences, completeness;
   - Vector spaces; normed spaces; continuity; uniform continuity; convergence; limits; Banach spaces; linear operators; functionals; compactness; examples of important function spaces.
   - Proof of Weirerstrass theorem
     - Reference: Kreyszig [14]
4. Calculus of Variation (2 Lectures)
   - Basic problem; Weak and strong extrema;
   - Euler-Lagrange equations;
   - Variable end-point problem;
   - Integral constraints; Second order conditions
     - Reference: Liberzon [11]
5. Variational approach to optimal control (2 Lectures)
   - Weierstrass-Erdmann corner condition
   - Basic problem formulation

    - Variational Approach to (i) fixed-time, free end-point and (ii) free-time, free end-point problems
    - Weakness of the variational approach
    - Statement of Pontryagin of Maximum Principle
        - ❖ Reference: Liberzon [11]
6. LQR , Bang-Bang control, singular control and other optimal control problems (5 Lectures)
        - ❖ Reference: Lewis [15] and Bryson [7]
7. Elements of Dynamical Systems (1 Lecture)
    - Definition of a solution; semi-group properties
    - Heine-Borel thm; Existence and uniqueness
    - Continuous dependence on initial data and parameters
    - ODE theorems needed for the proof of PMP
        - ❖ Reference: Murray [17]
8. Proof of Pontryagin Maximum Principle (4 Lectures)
        - ❖ Reference: Liberzon [11] and Evans [1]
9. Examples from Trajectory Optimization (2-4 Lectures)
10. Dynamic Programming
    - Derivation of HJB equations
        - ❖ Reference: Liberzon [11] and Evans [1]
    - Examples: rocket railroad car; general Linear Quadratic Regulator
    - Method of Characteristics and relationship to Pontryagin Maximum Principle
        - ❖ Reference: Liberzon [11] and Evans [1]
11. Stochastic Optimal Control
    - Review of relevant probability theory
    - Ito's Lemma
        - ❖ Reference: Mao [16]
    - Stochastic dynamic programming
        - ❖ Reference: Evans [1]

Grading

Grading will be determined by the best four of five HW assignments (60%) and one closed-book final examination (40%). It is highly encouraged to work together on the HW assignments. Each student, however, must turn in his or her HW individually. Some HWs may involve programming in Matlab.

Required Textbooks

- Liberzon, D. (2012). *Calculus of variations and optimal control theory: A concise introduction*. Princeton, N.J: Princeton University Press.

Recommended References

1. Evans, An Introduction to Mathematical Optimal Control Theory Version 0.2
2. Pinch, Enid R. *Optimal Control and the Calculus of Variations*. Oxford: Oxford University Press, 1995.
3. Bryson, A. E., & Ho, Y.-C. (1975). Applied optimal control: Optimization, estimation, and control. Washington: Hemisphere Pub. Corp.

4. Gelfand, I. M., & Fomin, S. V. (1963). *Calculus of Variations*. (P. L. García, A. Pérez-Rendón, & J. M. Souriau, Eds.)*America* (J. Vol. 1, p. 1). Cambridge Univ Pr.
5. John C. Amazigo, Lester A. Rubenfeld. Advanced calculus and its applications to the engineering and physical sciences.
6. Anderson and Moore, *Optimal Control*, Dover, 2007
7. A. Bryson, *Applied Linear Optimal Control*, Cambridge University Press, 2002
8. M. Athans and P. Falb, *Optimal Control*, Dover, 2006
9. D. Naidu, *Optimal Control Systems*, CRC Press, 2002
10. Mesterton-Gibbons, M. (2009). *A Primer on the Calculus of Variations and Optimal Control Theory*. *Journal of the Franklin Institute* (Vol. 21, p. 252). American Mathematical Society.
11. Liberzon, D. (2012). *Calculus of variations and optimal control theory: A concise introduction*. Princeton, N.J: Princeton University Press.
12. E. J. McShane. The calculus of variations from the beginning through optimal control theory. 27(5):916–939, 1977.
13. A history of the calculus of variations from the 17th through the 19th century, by Herman H. Goldstine, Studies in the History of Mathematics and Physical Sciences, vol. 5, Springer-Verlag, New York-Heidelberg-Berlin, 1980.
14. Kreyszig, E. (1989). Introductory Functional Analysis with Applications. SIAM Review (Vol. 21, p. 412). Wiley. Retrieved from http://link.aip.org/link/SIREAD/v21/i3/p412/s1&Agg=doi
15. F.L. Lewis, D. Vrabie, and V.L. Syrmos. Optimal Control. John Wiley & Sons, 2012.
16. X. Mao, Stochastic Differential Equations and Applications, Horwood, Chichester, 1997
17. Francis J. Murray and Kenneth S. Miller, Existence Theorems for Ordinary Differential Equations, Dover Books on Mathematics, 2007

Optimal Control

Schedule for: ECE 18879K: Special Topics in Systems and Control; Fall 2012 Total Classes: 28

	Class Meets			*Topic*	*Reading*	*Deadlines*
	Tues	*Thurs*	*Fri*			
Wk 1	8/28	8/30	8/31	Historical perspectives, basic notions and motivating examples	Notes; Ch 1 (Lib)	
Wk 2	9/4	9/6	9/7	Dimensional Spaces	Notes; Ch 1 (Lib)	HW #1 Due 9/7
Wk 3	9/11	9/13	9/14	Some Elements of Functional Analysis; Calculus of Variation	Notes;	
Wk 4	9/18	9/20	9/21	Calculus of Variation; Variational approach to optimal control	Notes; Ch 2 (Lib)	HW #2 Due 9/21
Wk 5	9/25	9/27	9/28	Variational approach to optimal control	Notes; Ch 3 (Lib)	
Wk 6	10/2	10/4	10/5	LQR , Bang-Bang control, singular control and other optimal control problems	Notes; Ch 3 (Lewis)	
Wk 7	10/9	10/11	10/12	LQR , Bang-Bang control, singular control and other optimal control problems	Notes; Ch 3 (Lewis)	HW #3 Due 10/12
Wk 8	10/16	10/18	10/19	Elements of Dynamical Systems	Notes;	
Wk 9	10/23	10/25	10/26	Proof of Pontryagin Maximum Principle	Notes; Ch 4 (Lib)	
Wk 10	10/30	11/1	11/2	Proof of Pontryagin Maximum Principle	Notes; Ch 4 (Lib)	HW #4 Due 11/2
Wk 11	11/6	11/8	11/9	Examples from Trajectory Optimization	Notes;	
Wk 12	11/13	11/15	11/16	Examples from Trajectory Optimization	Notes;	
Wk 13	11/20	11/22	11/23	Dynamic Programming	Notes; Ch 5 (Lib)	Thanksgiving
Wk 14	11/27	11/29	11/30	Derivation of HJB	Notes; Ch 5 (Lib)	
Wk 15	12/4	12/6	12/7	Stochastic Optimal Control; Ito Calculus	Notes;	HW #5 Due 12/7
Wk 16	12/11	12/13	12/14			Exam Week
Wk 17	12/18	12/20	8/31			Grades due on Dec. 20

Breakdown:			
HW 1			15%
HW 2			15%
HW 3			15%
HW 4			15%
HW 5			15%
Total HW			60%
Final			40%
Total			**100%**

No Class ▨

Due Date 🟨

You can drop one HW; best 4 of 5 (total 60%)
Final - 40%