Automatica 49 (2013) 1012–1018

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Tube model reference adaptive control[☆]

Boris Mirkin[1], Per-Olof Gutman

Faculty of Civil and Environmental Engineering, Technion – Israel Institute of Technology, Haifa 32000, Israel

ARTICLE INFO

Article history:
Received 13 February 2012
Received in revised form
21 October 2012
Accepted 31 October 2012
Available online 22 February 2013

Keywords:
Robust model reference adaptive control
Optimization

ABSTRACT

By using the concept of on-line goal adaptation, we develop a new paradigm of performance shaping in MRAC. The general idea is to replace the single reference model generated trajectory in classical adaptive design with a tube reference model. Two alternative adaptive control schemes that lead to tractable design formulations are developed in which the performance is adapted on-line to satisfy a new specification in addition to maintaining the usual stability and robustness properties. For this purpose an additional optimization problem is formulated within the MRAC framework to find a correction control term at each instant of time. The proposed approach provides a convenient intuitive interpretation of the design problem, while retaining the fundamental ideas on which model reference adaptive control is based. The system performance is found to be as desired by simulation.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

Model reference adaptive control (MRAC) is one of the main approaches of adaptive control, see e.g. the popular textbooks (Åström & Wittenmark, 1995; Ioannou & Sun, 1996; Krstić, Kanellakopoulos, & Kokotović, 1995; Narendra & Annaswamy, 1989; Sastry & Bodson, 1989; Tao, 2003). Most model reference adaptive control design techniques have paid attention only to control problem solutions for one particular performance index — the tracking error which is the difference between the plant output and the reference model output. The reference model is chosen to generate a *single desired trajectory* that the plant output has to follow. Choosing a single reference trajectory is an idealization in order to obtain a solution based on the relevant mathematics.

In many applications, such as industrial process control or flight control, it is not necessary to exactly follow a single reference trajectory; usually some specified deviation is allowed. Several approaches can be taken when a control specification is given in the form of an admissible set, and interesting problems can be posed. Here, we mention only some works in the context of adaptive control: so called "funnel control" in Ilchman, Ryan, and Townsend (2007) where approximate tracking and prescribed transient behaviour of the tracking error are both captured by the concept of a performance funnel; a reference model chosen to be piecewise linear e.g. Sang and Tao (2010); and an on-line adaptation of the reference model as is demonstrated in Joshi, Tao, and Patre (2011) when the plant-model matching conditions are violated whereby the adaptive controller includes tuning of the controller gain and simultaneous estimation of the plant-model mismatch.

In the framework of the concept of goal adaptation, and with the aim to provide additional desirable properties of the closed-loop system, see Mirkin (2001) and Mirkin and Mirkin (1999), a new paradigm of performance shaping in MRAC, called Tube based Model Reference Adaptive Control (TMRAC) was recently developed in Mirkin, Gutman, and Shtessel (2012); Mirkin, Gutman, and Sjöberg (2011). It is advised to design a controller which not only guarantees closed loop stability, asymptotical tracking, and robustness to various uncertainties in the plant model and to external disturbances, but also diminishes the control cost with respect to some criterion. For this the control signal is split into two parts – an adaptive component and a component that corrects the control objective – and an additional optimization problem is formulated in order to find the value of the newly defined correction control component in each time instant.

In the context of TMRAC we develop here two alternative adaptive control schemes that lead to tractable design formulations where the performance is adapted on-line to satisfy additional requirements. The problem is treated from two different viewpoints: when the correcting component is included in the so-called "regressor" vector, and when it is treated as an external bounded perturbation.

2. Statement of the MRAC problem with performance tube

Before proceeding to the main results, we describe the principal idea of tube reference model based adaptive control

☆ This work was supported by the Israel Science Foundation under Grant 588/07. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Gang Tao under the direction of Editor Miroslav Krstic.

E-mail addresses: bmirkin@technion.ac.il (B. Mirkin), peo@technion.ac.il (P.-O. Gutman).

[1] Tel.: +972 48320974; fax: +972 48295696.

0005-1098/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2013.01.022

with on-line cost adaptation (Mirkin et al., 2012, 2011; Mirkin & Mirkin, 1999). As in standard adaptive control theory, the desired performance of the controlled system, the so called goal objective, is expressed with the help of a reference model which gives the desired response to a command signal.

However the reference will not be given, as usual, as a unique trajectory, but as *any trajectory from some admissible set*, i.e. the general concept is to replace the classical adaptive goal in the form of a reference model with a single predetermined trajectory with *a tube reference model*. Then, in addition to the usual stability and robustness requirements, other important specifications may be considered. For comparison we note that in classical robust linear time invariant control (Horowitz, 1993), tighter reference trajectory specifications demand higher bandwidth from the feedback loop.

Let us make it possible to change, within specified bounds, the input signal to the reference model, or some parameters of the reference model operator W_r. Let us call these input signals and parameters *goal correction control* u_c. By tuning u_c within the allowed bounds, an admissible set of reference trajectories is generated and is called the *performance tube*. The correction control bounds are specified such that the trajectories defined by the performance tube satisfy the closed loop system specifications. In addition to influencing the reference model, the goal correction control u_c is added to the feedback control signal. Based on measurements of the system signals, the adaptive control algorithm chooses on-line a suitable trajectory from the performance tube by varying u_c.

The main problem is *how to determine a formal mechanism to choose reference trajectories from the performance tube*. For that purpose, we formulate here the goal in the form of two control objectives — primary and secondary: (i) the *primary* goal is to make the system states/outputs asymptotically follow the state/output of a stable reference model; and (ii) the *secondary* goal is to satisfy some additional criterion by varying the goal correction control $u_c(t)$ within its given specified limits. Finally it is shown that both of these problems can be solved independently of one another. Thus the defined problem of model reference adaptive control is solved in two stages.

2.1. The first stage design problem

The design problem in the first stage is to find a control law parametrization in order to ensure closed-loop stability for any input to the reference model, and *asymptotical model tracking* for the feasible trajectories. In this design stage only the structure of the reference model is known, with some variables defined incompletely which will, in the next stage, be used as design variables which may vary within assigned limits. The solution of the first stage design problem is necessary for the second stage.

2.2. The second stage design problem

In this stage we seek a mechanism to adjust the goal correction control u_c in order to receive certain additional beneficial properties in the closed loop system. The additional performance index can include decreasing control costs, control amplitude constraints, etc. In this paper the additional performance index is a function of the control signals.

3. TMRAC. State feedback case

To facilitate the introduction of the principal concepts contained in this paper, we start our discussions with a relatively simple adaptive control problem whose solution can be found in any standard text on adaptive control. In this section, we consider the adaptive control of a linear time-invariant plant with parametric uncertainty, when the state variables of the plant are measured.

3.1. Plant model

The plant is described by the state equation

$$\dot{x}(t) = Ax(t) + bu(t) \tag{1}$$

with $A \in \mathbb{R}^{n\times n}$ and $b \in \mathbb{R}^n$ being unknown and constant parameter matrices, and $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^1$ being the system state and input signals.

3.2. Control objectives

3.2.1. The primary control specification

The primary control specification is to determine the input to the plant $u(t)$ such that all signals in the closed-loop system become bounded and $x(t)$ asymptotically exactly tracks the given reference signal $x_r(t)$ which is generated from the reference model

$$\dot{x}_r(t) = A_r x_r(t) + b_r u_r(t), \quad u_r(t) = r(t) + u_c(t) \tag{2}$$

where $A_r \in \mathbb{R}^{n\times n}$ is a stable matrix, $b_r \in \mathbb{R}^n$, $r(t)$ is in the form of a standard reference signal of adaptive control theory (Ioannou & Sun, 1996; Tao, 2003), and the goal correction component of the command signal $u_c(t)$ may vary within specified given limits

$$u_c(t) \in [u_c^-(t)\ u_c^+(t)] \tag{3}$$

which determine the performance tube of the reference model (2).

3.2.2. The secondary control objective

The secondary control objective is formalized as the following optimization task:

$$\begin{aligned} &\textit{minimize}(w.r.t.\ u_c) \quad J\big(u_c\big) = u^2(t) \\ &\textit{subject to} \quad u_c^-(t) \le u_c(t) \le u_c^+(t). \end{aligned} \tag{4}$$

3.2.3. Assumptions

As is generally done in traditional MRAC, we assume that there exist a constant vector $\theta_x^* \in \mathbb{R}^n$ and a nonzero constant scalar θ_r^* such that the following equations are satisfied,

$$A + b\theta_x^{*T} = A_r, \qquad b\theta_r^* = b_r \tag{5}$$

and the sign of θ_r^* is known. Without loss of generality, we will assume that θ_r^* is positive.

3.3. Control parametrization and the basic error equation

3.3.1. Controller structure

The control $u(t)$ is proposed to be the sum of two signals — the adaptive signal $u_a(t)$, and goal correction signal $u_c(t)$

$$u(t) = u_a(t) + u_c(t), \tag{6}$$

as illustrated in the control system block diagram in Fig. 1.

3.3.2. Error equation and adaptive control design

In view of (1), (2), (5) and (6) the tracking error $e(t) = y(t) - y_r(t)$ for any $u(t)$ can be expressed as

$$\begin{aligned} \dot{e}(t) &= A_r e(t) + b[u(t) - \theta_x^{*T} x(t) - \theta_r^* u_r(t)] \\ &= A_r e(t) + b[u_a(t) - \theta_x^{*T} x(t) - \theta_r^* r(t) - \theta_c^* u_c(t)] \\ &= A_r e(t) + b\big[u_a(t) - \theta^{*T}\omega(t) - \theta_c^* u_c(t)\big] \end{aligned} \tag{7}$$

where $\omega(t) = [x(t)\ r(t)]^T \in \mathbb{R}^{n+1}$, $\theta^* = [\theta_x^{*T},\ \theta_r^*]^T \in \mathbb{R}^{n+1}$ and $\theta_c^* = \theta_r^* - 1$.

The terms $\theta^{*T}\omega(t)$ and $\theta_c^* u_c(t)$ in (7) suggest that one should look for a control law $u_a(t)$ parametrization such that the

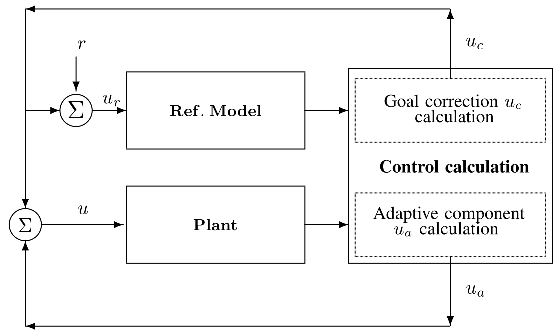

Fig. 1. Control setup.

adaptation gains enter as a product in which the factors $\omega(t)$ and $u_c(t)$ are known signals ("regressor"), as is usually done in adaptive control theory,

$$u_a(t) = \theta^T(t)\omega(t) + \theta_c(t)u_c(t). \quad (8)$$

The design task then is to choose suitable adaptive laws to update the vector gain $\theta(t)$ and the scalar gain $\theta_c(t)$ such that the control objective is achieved, i.e. $e(t) \to 0$ when $t \to \infty$. To develop adaptive laws for $\theta(t)$ and $\theta_c(t)$ we need, as usual, an error equation in terms of the tracking error $e(t)$ and the parameter errors $\tilde{\theta}(t) = \theta(t) - \theta^*$ and $\tilde{\theta}_c(t) = \theta_c(t) - \theta_c^*$. So we obtain from (7) the basic error equation

$$\dot{e}(t) = A_r e(t) + b\tilde{\theta}^T(t)\omega(t) + b\tilde{\theta}_c(t)u_c(t). \quad (9)$$

With the model (9) it is possible to apply the well-known Lyapunov based technique see, e.g. Ioannou and Sun (1996); Tao (2003) to design updating laws for the gains $\theta(t)$ and $\theta_c(t)$, and to prove stability.

The following theorem gives one of the possible choices of adaptation laws that ensure asymptotic state tracking with signal boundedness.

Theorem 1. *The system given by (1), (2), the adaptive controller (6), (8), and the gain adaptation laws*

$$\dot{\theta}(t) = -\Gamma_I\eta(t) - \Gamma_p\dot{\eta}(t), \quad \eta(t) = S(t)\omega(t), \; S(t) = e^T(t)Pb_r$$
$$\dot{\theta}_c(t) = -\gamma_c S(t)u_c(t) + \theta_{cpr}(t) \quad (10)$$

where

$$\theta_{cpr}(t) = \begin{cases} 0, & \text{if } |\theta_c + 1| \geq \epsilon \text{ or} \\ & \text{if } |\theta_c + 1| = \epsilon \text{ and } S(t)u_c \leq 0; \\ \gamma_c S(t)u_c(t), & \text{otherwise,} \end{cases} \quad (11)$$

is the projection function, $\Gamma_I = \Gamma_I^T > 0$, $\Gamma_p = \Gamma_p^T > 0$, $\gamma_c > 0$ are the design parameters, ϵ is a known lower bound for $|\theta_c^* + 1| = |\theta_r^*| = \theta_r^*$, and P is the solution of the Lyapunov equation

$$A_r^T P + PA_r + Q = 0, \quad Q = Q^T > 0 \quad (12)$$

guarantee that all closed-loop signals are bounded, and the tracking error $e(t) = x(t) - x_r(t)$ goes to zero asymptotically for any $u_c(t)$ from (3).

Remark 1. As will be seen later, when solving the optimization problem (4), the adaptive gain $\theta_c(t)$ has a point of singularity. Therefore in order to prevent $\theta_c(t)$ from taking the singularity value, parameter projection is used in the adaptive controller for updating the gain $\theta_c(t)$.

Remark 2. For stability and exact asymptotic tracking, only the integral component $-\Gamma_I\eta(t)$ in (10) of the adaptation algorithm for $\theta(t)$ is needed. However, for the adaptation of $\theta(t)$ a proportional-integral (PI) or a proportional-integral-time delay (PITD) type algorithm can also be chosen which may lead to better transient adaptation performance than the traditional I scheme, see e.g. Mirkin and Gutman (2010). The sketch of the proof below is presented for the case of PI adaptation.

A sketch of the proof. Define

$$V = e^T(t)Pe(t) + \frac{1}{\theta_r^*}\left[\tilde{\eta}^T(t)\Gamma_I^{-1}\tilde{\eta}(t) + \gamma_c^{-1}\tilde{\theta}_c^2(t)\right] \quad (13)$$

where $\tilde{\eta}(t) = \tilde{\theta}(t) + \Gamma_p\eta(t)$.

To obtain the time derivative $\dot{V}$, we at first note that, in view of (10), the time derivative of $\tilde{\eta}(t)$ from (13) is

$$\dot{\tilde{\eta}}(t) = \dot{\tilde{\theta}}(t) + \Gamma_p\dot{\eta}(t) = -\Gamma_I\eta(t).$$

Then, using the last expression and (12), we have

$$\begin{aligned}\dot{V} &= -e^T(t)Qe(t) + \frac{2}{\theta_r^*}\tilde{\theta}^T(t)\eta(t) + \frac{2}{\theta_r^*}\tilde{\theta}_c(t)S(t)u_c(t) \\ &\quad - \frac{2}{\theta_r^*}\left[\tilde{\theta}(t)\eta(t) + \eta^T(t)\Gamma_p\eta(t) + \tilde{\theta}_c(t)S(t)u_c(t)\right] \\ &\quad + \frac{2}{\gamma_c\theta_r^*}\tilde{\theta}_c(t)\theta_{cpr}(t) \\ &= -e^T(t)Qe(t) - \eta^T(t)\Gamma_p\eta(t) + \frac{2}{\gamma_c\theta_r^*}\tilde{\theta}_c(t)\theta_{cpr}(t). \end{aligned} \quad (14)$$

The choice of the projection function $\theta_{cpr}(t)$ from (28) ensures that $\tilde{\theta}_c(t)\theta_{cpr}(t) \leq 0$ see e.g. Ioannou and Sun (1996, p. 400), i.e.

$$\dot{V} \leq 0 \quad (15)$$

which indicates that the desired stability properties of the closed-loop system are obtained, following the usual arguments of adaptive control theory.

3.3.3. *General comments*

Considering basic error equation (9) one can make the following observations:

(i) the first fundamental observation here is that the basic tracking error equation that is generally used for stability analysis, and for the design of the adaptation algorithm, has the standard "classical" so-called bilinear parametric model form, see e.g. Ioannou and Sun (1996); Tao (2003). It *does not explicitly depend on the goal correction component* $u_c(t)$ of the command signal, and, therefore, the latter *can be changed*;
(ii) using this fact, the problem of ensuring the primary ($e(t) \to 0$) and the secondary control objectives (4) can be solved independently of each other, and the control components $u_a(t)$ and $u_c(t)$ in (6) can be synthesized separately. So, by proving stability by synthesizing $u_a(t)$ independently of any $u_c(t)$ from (3), the approach used in this paper *decouples the two problems and permits the optimization to be carried out independently*;
(iii) the regressor vectors have the traditional form except that they include the additional signal $u_c(t)$;
(iv) to assure the stability of the error model, various modifications of existing adaptive laws can be chosen, based on the available MRAC and robust MRAC techniques.

3.3.4. *Correction control component design*

As established in Theorem 1, the closed-loop systems stability with $\lim_{t\to\infty} e(t) = 0$ is guaranteed for any choice of feasible $u_c(t)$ from (3). Its variability does not change the stability properties, and therefore does not affect the achievement of the primary goal. Both control goals can be achieved *independently*. Of course, the solution of the first problem is the key without which the second task cannot be solved. Keeping this in mind, it is easy to show that the optimization problem (4) with the index $J = u^2 = (u_a + u_c)^2$, and with $u_a(t) = \theta^T(t)\omega(t) + \theta_c(t)u_c(t)$ from (8), (10) and for any bounded $\theta^T(t)$, $\omega(t)$ and $\theta_c(t)$ has the following solution

$$u_c^{opt} = \begin{cases} -u_a(t), & \text{if } u_a(t) \in [u_c^- \ u_c^+]; \\ \arg\min\Big(J(u_c^-),\ J(u_c^+)\Big), & \text{if } u_a(t) \notin [u_c^- \ u_c^+] \end{cases} \tag{16}$$

where $J(u_c^-)$ and $J(u_c^+)$ are the values of $J = (u_a(t) + u_c(t))^2$ for $u_c(t) = u^-$ and $u_c(t) = u^+$, respectively.

Hence, the controller (6), (8), (16): (i) guarantees that the tracking errors tend to zero; and (ii) minimizes the control cost.

Remark 3. Note that the solution of the optimization problem (4), in the case when $u_a(t) \in [u_c^- \ u_c^+]$, has the form $u_c^*(t) = -(1 + \theta_c(t))^{-1}\theta^T(t)\omega(t)$. So the adaptive gain $\theta_c(t)$ has a point of singularity. In order to prevent $\theta_c(t)$ from taking the value -1, we suggest using e.g. the parameter projection algorithm in Ioannou and Sun (1996, p. 400) for updating the gain $\theta_c(t)$.

Remark 4. Note, that in some cases the bounds u_c^+ and u_c^- can also be chosen. Such an approach is well adapted to computer aided design. If the design result is not satisfactory, the designer can iteratively test modifications of the limits to satisfy the desired specification. This is very attractive, e.g. for reducing the control effort.

3.4. Example 1

We conclude this section by considering a simulated example of TMRAC control of a simple scalar plant. The unstable plant to be controlled is

$$\dot{y}(t) = ay(t) + bu(t), \qquad y(0) = y_0, \tag{17}$$

and the reference model is

$$\dot{y}_r(t) = a_r y_r(t) + b_r u_r(t), \quad u_r(t) = r(t) + u_c(t). \tag{18}$$

The adaptive controller from (6), (8), (10) and (28), with the goal correction control signal $u_c(t)$ from (16), and the secondary control objective $J = u^2 = (u_a + u_c)^2$ from (4) is

$$\dot{\theta}(t) = -\Gamma_I \eta(t) - \Gamma_p \dot{\eta}(t), \quad \eta(t) = \omega(t)e(t)$$

$$\dot{\theta}_c(t) = -\gamma_c e(t) u_c(t) + \theta_{cpr}(t) \tag{19}$$

where $\omega^T(t) = [y(t), r(t)]$, $\theta(t) = [\theta_y(t)\theta_r(t)]^T \in \mathbb{R}^2$ and $\theta_{cpr}(t)$ is from (28) with $S(t) = e(t)$. The parameter values are chosen as $a = 1$, $b = 0.5$, $a_r = -5$, $b_r = 2$, $u_c^+ = 1$, $u_c^- = -1$, $\Gamma_I = 9$, $\Gamma_p = 0.6$. In this example all the plant parameters are unknown to the controller. The input signal to the reference model $r(t)$ is a step signal with amplitude ± 3. Some simulation results are shown in Figs. 2–4. For comparison of performance with the conventional scheme (i.e. $u_c = 0$ and $\theta_c = 0$ in (18), (19)), the input signal to the reference model r was adjusted to reach the same steady state reference trajectory, and therefore the same steady state control signal. All other controller parameters remained the same. As can be seen from the lower graphs in Figs. 2 and 3, control based on the tube reference model results in improved performance compared to conventional control.

4. TMRAC. Output feedback case

Also for adaptive control by output feedback, it is possible to get the form of the basic error equation explicitly independent of the goal correction signal $u_c(t)$.

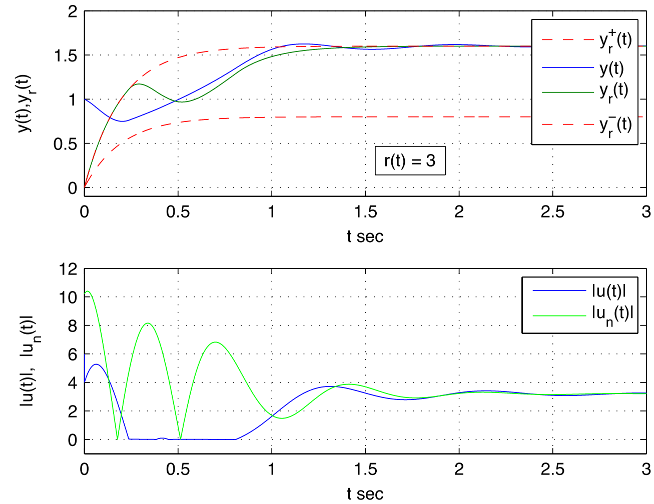

Fig. 2. Simulation of the adaptive control system for plant (17) and controller (6). The upper graphs show how the adaptive controller finds a reference trajectory within the admissible range $y_r^- \le y_r(t) \le y_r^+$. The lower graphs show the time history of the absolute value of the control signals $|u(t)|$ for the cases of adaptive control with (blue lines) and without (green lines) optimization, respectively.

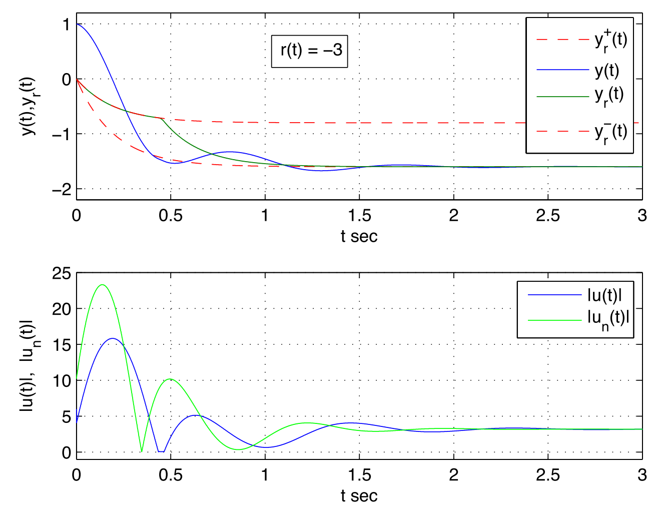

Fig. 3. Simulation of the adaptive control system for plant (17) and controller (6). The upper graphs show how the adaptive controller finds a reference trajectory within the admissible range $y_r^- \le y_r(t) \le y_r^+$. The lower graphs show the time history of the absolute value of the control signals $|u(t)|$ for the cases of adaptive control with (blue lines) and without (green lines) optimization, respectively.

4.1. Plant model

Let us consider the SISO plant given by Ioannou and Sun (1996, Section 6.4.1)

$$y = W_o(s)u, \quad W_o(s) = k_p \frac{N(s)}{D(s)} \tag{20}$$

where $u(t)$ and $y(t)$ are plant inputs and output respectively.

4.2. Control objectives

4.2.1. The primary control specification

The primary control specification is to design an output feedback control signal $u(t)$ such that all signals of the closed-loop system are bounded, and the output signal $y(t)$ asymptotically exactly tracks the output of the reference model $y_r(t)$ given by

$$y_r(t) = W_r(s)u_r = k_r \frac{N_r(s)}{D_r(s)} u_r, \qquad u_r(t) = r(t) + u_c(t) \tag{21}$$

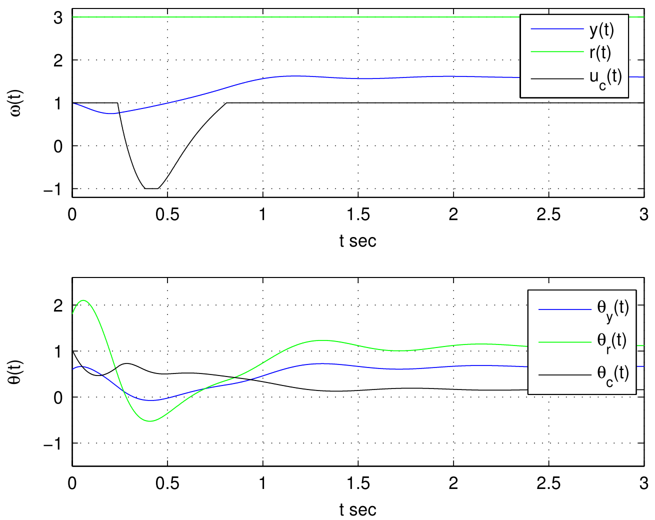

Fig. 4. Simulation of the adaptive control system for the plant (17) and the controller (6), for the case when $r(t) = 3$. The upper and lower graphs show the time history of the regressor $\omega(t)$ and the adaptive gain $\theta(t)$ vectors, respectively.

where $r(t)$ is a standard reference signal of adaptive control theory (Ioannou & Sun, 1996; Tao, 2003), and the goal correction component $u_c(t)$ of the command signal may vary within its specified given limits (3).

4.2.2. The secondary control objective

The secondary control objective is formalized as the optimization task from (4).

4.3. Assumptions

The following standard assumptions Ioannou and Sun (1996); Tao (2003) are made for the plant and reference model: (A1) $D(s)$ is a monic polynomial of known degree n, (A2) $W_o(s)$ is minimum phase, i.e. $N(s)$ is Hurwitz, (A3) the relative degree is one and (A4) the sign of the high frequency gain k_p is known.

Remark 5. The minimum phase assumption (A2) is fundamental in MRAC schemes. Assumption (A3) focuses on the simplest case amenable to Lyapunov designs to demonstrate the main idea in this contribution.

4.4. Error equation parametrization

By using the conventional technique of model reference adaptive control (Ioannou & Sun, 1996; Tao, 2003) the tracking error $e(t) = y(t) - y_r(t)$ for any $u(t)$ can be expressed as

$$e = W_r(s)\rho^*\left[u - \theta_y^* y - \theta_1^{*T} x_1 - \theta_2^{*T} x_2 - \theta_r^* u_r\right] \tag{22}$$

where

$$x_1 \in \mathbb{R}^{n-1} = H_f(s)[u], \qquad x_2 \in \mathbb{R}^{n-1} = H_f(s)[y],$$

$$H_f(s) = \frac{[s^{n-2} \ \ldots \ s \ 1]^T}{\lambda(s)}, \quad H_f(s) \in \mathbb{R}^{n-1}$$

$\lambda(s) = s^{n-1} + \cdots + \lambda_1 s + \lambda_0$ is a monic Hurwitz polynomial; $\rho^* = \theta_r^{*-1} = k_p k_r^{-1}$ and $\theta_1^*, \theta_2^* \in \mathbb{R}^{n-1}$, $\theta_y^* \in \mathbb{R}$, $\theta_r^* \in \mathbb{R}$ are so-called matching parameters.

4.5. Controller structure and basic error model

As follows from the TMRAC concept, see Section 2–3, the control signal $u(t)$ is decomposed as the sum of two components — an adaptive $u_a(t)$ and a goal correction component $u_c(t)$,

$$u(t) = u_a(t) + u_c(t). \tag{23}$$

Applying this control to (22), the error model (22) can now be rewritten as

$$\begin{aligned} e &= W_r(s)\rho^*\left[u_a - \theta_y^{*T} y - \theta_1^{*T} x_1 - \theta_2^{*T} x_2 - \theta_r^* r - \theta_c^* u_c\right] \\ &= W_r(s)\rho^*\left[u_a - \Theta^{*T}\Omega(t)\right] \end{aligned} \tag{24}$$

where $\theta_c^* = \theta_r^* - 1$ and

$$\Omega(t) = [y(t), x_1(t), x_2(t), r(t), u_c(t)]^T = [\omega^T(t), u_c(t)]^T$$
$$\Theta^* = [\theta_y^*, \theta_1^{*T}, \theta_2^{*T}, \theta_r^*, \theta_c^*]^T = [\theta^{*T}, \theta_c^*]^T.$$

Then the following parametrization for the adaptive component of (23)

$$u_a(t) = \theta^T(t)\omega(t) + \theta_c(t)u_c(t) \tag{25}$$

with the adaptation gains

$$\theta(t) = [\theta_y(t),\ \theta_1^T(t),\ \theta_2^T(t),\ \theta_r(t)]^T, \quad \theta_c(t)$$

defines the basic error equation for the output feedback case

$$\begin{aligned} e &= W_r(s)\rho^*\widetilde{\Theta}^T(t)\Omega(t) \\ &= W_r(s)\rho^*\left(\tilde{\theta}^T(t)\omega(t) + \tilde{\theta}_c(t)u_c(t)\right) \end{aligned} \tag{26}$$

where $\widetilde{\Theta}(t)$ is the parameter error vector.

Remark 6. We note that all the observations we made in Section 3.3.3 hold also in this output feedback case, and the unified form of the basic tracking error equations makes it possible to synthesize the adaptive control laws by all methods.

4.6. Design of the adaptation laws

For output tracking the following theorem gives one of the possible choices of adaptation laws that ensures asymptotic output tracking with signal boundedness.

Theorem 2. *Consider system (20) and the reference model (21). Suppose that assumptions of Section 4.3 hold. Then the control (25) with update laws*

$$\begin{aligned} \dot{\theta}(t) &= -\Gamma_I\eta(t) - \Gamma_p\dot{\eta}(t), \quad \eta(t) = e(t)\omega(t) \\ \dot{\theta}_c(t) &= -\gamma_c e(t)u_c(t) + \theta_{cpr}(t) \end{aligned} \tag{27}$$

where

$$\theta_{cpr}(t) = \begin{cases} 0, & \text{if } |\theta_c + 1| \geq \epsilon \ \text{ or} \\ & \text{if } |\theta_c + 1| = \epsilon \ \text{ and } \ S(t)u_c \leq 0; \\ \gamma_c e(t)u_c(t), & \text{otherwise.} \end{cases} \tag{28}$$

guarantees that (i) *all signals of the closed-loop system are bounded and* (ii) $\lim_{t\to\infty} e(t) = 0$ *for any the goal correction signal* $u_c(t)$ *from (3).*

The proof of this theorem, i.e. proving the attainment of the primary objective, follows along the same lines as e.g. in the textbooks (Ioannou & Sun, 1996; Tao, 2003) and Section 3.

As established in Theorem 2, the closed-loop system stability with $\lim_{t\to\infty} e(t) = 0$ is guaranteed for any $u_c(t)$ from (3). Then it is easy to show that the optimization problem (4) with u from (23), (27) has the solution described in Section 3.3.4, i.e. the adaptive controller (23), (25) and (27) guarantees the primary and secondary design objectives.

4.7. Example 2

The following problem was simulated to illustrate the output feedback TMRAC. The second order plant with parametric uncer-

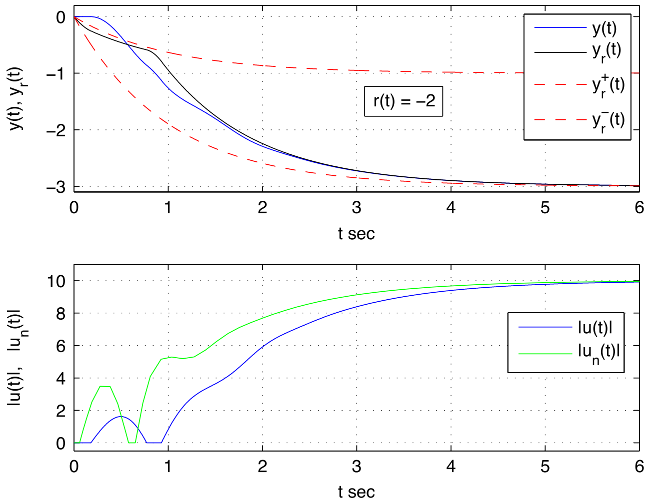

Fig. 5. Simulation of the adaptive control system for plant (29) and controller (23). The upper graphs show how the adaptive controller finds a reference trajectory within the admissible range $y_r^- \leq y_r(t) \leq y_r^+$. The lower graphs show the time history of the absolute value of the control signals $|u(t)|$ for the cases of adaptive control with (blue lines) and without (green lines) optimization, respectively.

tainties and the suitable reference model are chosen from Ioannou and Sun (1996, Example 6.4.1)

$$y = \frac{k_p(s+b_0)}{s^2+a_1s+a_0}u, \qquad y_r = \frac{1}{s+1}u_r \tag{29}$$

where $k_p > 0$, $b_0 > 0$ and k_p, b_0, a_1, a_0 are unknown constants.

The simulation results for the plant (29) and the adaptive controller (23), (25)

$$\begin{aligned}\dot{x}_1(t) &= -2x_1(t) + u(t)\\ \dot{x}_2(t) &= -2x_2(t) + y(t)\\ u_a(t) &= \theta^T(t)\omega(t) + \theta_c(t)u_c(t)\\ \theta(t) &= [\theta_y(t)\ \theta_1^T(t)\ \theta_2^T(t)\ \theta_r(t)]^T\end{aligned} \tag{30}$$

with adaptive laws from (27) and $u_c(t)$ from (16) are shown in Fig. 5. The parameter values in our simulation were chosen as $b_0 = 3$, $a_1 = 3$, $a_0 = -10$, $k_p = 1$, $\Gamma_I = 5$, $\Gamma_p = 0.5$, $u_c^+ = 1$, $u_c^- = -1$ and $r = -2$.

As in Example 1 the picture shows how the adaptive controller finds a desirable reference trajectory within the admissible range and improves performance compared to the traditional control scheme. Note also that simulation examples with external disturbances and nonlinear perturbations can be found in Mirkin et al. (2012, 2011).

4.8. Robustness issues

In the previous sections we studied adaptive control for plants with parametric uncertainties, only. Now we briefly show that the use of the new performance shaping makes it possible to also achieve a robust design for plants with external disturbances and un-modelled dynamics. We propose two possible control parametrizations.

Let us consider the SISO plant given by Ioannou and Sun (1996, Section 9.3)

$$y = W_o(s)\big(1 + \Delta_m(s)\big)(u + d), \tag{31}$$

where the transfer function of the modelled part of the plant $W_o(s)$ is the same as in (20), d is an external bounded disturbance, i.e. $|d(t)| \leq d_o$ and $\Delta_m(s)$ is the multiplicative un-modelled dynamics.

The primary control specification is now to design an output feedback control signal $u(t)$ such that all signals of the closed-loop system are bounded, and the output signal $y(t)$ tracks, *as closely as possible*, the output of the reference model $y_r(t)$ given by (21). As before *the secondary control objective* is formalized as the optimization task from (4).

In addition to assumptions in 4.3 the following assumptions are made for the un-modelled part of the plant (Ioannou & Sun, 1996, p. 673): (A5) $\Delta_m(s)$ is analytic in $Re[s] \geq -\delta_0/2$ for some known $\delta_0 > 0$. (A6) There exists a strictly proper transfer function $P(s)$ analytic in $Re[s] \geq -\delta_0/2$, and such that $P(s)\Delta_m(s)$ is strictly proper.

By using the conventional technique of robust model reference adaptive control (Ioannou & Sun, 1996, Section 9.3) the tracking error $e(t) = y(t) - y_r(t)$ for any $u(t) = u_a(t) + u_c(t)$ can be expressed as

$$e = W_r(s)\rho^*\big[u_a - \theta_y^{*T}y - \theta_1^{*T}x_1 - \theta_2^{*T}x_2 - \theta_r^*r - \theta_c^*u_c + \eta_0\big] \tag{32}$$

where all the notations are the same as in (22) and $\eta_o = H(s)\Delta_m(s)(u+d) + H(s)d$, with $H(s) = 1 - \theta_1^{*T}H_f(s)$.

The key observation. From this equation the following key observation can be made. Depending on how the bounded term $\theta_c^*u_c$ is handled in the error equation (32), two alternative schemes of controller parametrization may be proposed: **(i)** $u_c(t)$ is a component of the "regressor" vector, similarly to (24); **(ii)** $u_c(t)$ is treated as a bounded external disturbance, and therefore it can be handled the same way as e.g. the real bounded disturbance $d(t)$. This means that $u_c(t)$ can be absorbed in the bounded signal η_o during the control law synthesis.

Then two different error equations can be obtained from (32) in unified form,

$$\begin{aligned}&\textbf{(i)}: e = W_r(s)\rho^*\big[u_a - \theta_1^{*T}\omega_1 + \eta_1\big]\\ &\textbf{(ii)}: e = W_r(s)\rho^*\big[u_a - \theta_2^{*T}\omega_2 + \eta_2\big]\end{aligned} \tag{33}$$

and, as a consequence, two adaptive control schemes. In case **(i)**, the regressor ω and the unknown vector θ^* have to be chosen as

$$\begin{aligned}\omega(t) &= [y(t)\ x_1^T(t)\ x_2^T(t)\ r(t)\ u_c(t)]^T \in \mathbb{R}^{2n}\\ \theta^* &= [\theta_y^*\ \theta_1^{*T}\ \theta_2^{*T}\ \theta_r^*\ \theta_c^*]^T \in \mathbb{R}^{2n}\end{aligned}$$

and for case **(ii)**

$$\begin{aligned}\omega(t) &= [y(t)\ x_1^T(t)\ x_2^T(t)\ r(t)]^T \in \mathbb{R}^{2n-1}\\ \theta^* &= [\theta_y^*\ \theta_1^{*T}\ \theta_2^{*T}\ \theta_r^*]^T \in \mathbb{R}^{2n-1}.\end{aligned}$$

The expression for η_1 has the standard form i.e. $\eta_1 = \eta_0$ from (22), and the expression for η_2 is slightly modified to be $\eta_1 = H(s)\Delta_m(s)(u+d) + H(s)d - \theta_c^*u_c$.

Remark 7. We note that all the observations we made in Section 3.3.3 hold also in this general case, and the unified form of the basic tracking error equations makes it possible to synthesize the adaptive control laws by all methods. Note, however, that in the case of the error models in (33), the adaptive laws, in contrast to Theorem 2, will give tracking error convergence to some *bounded residual set*, only. But for a certain class of problems, design procedures described in Mirkin and Gutman (2010); Mirkin et al. (2012, 2011) make it possible to achieve asymptotical exact tracking by treating $u_c(t)$ as external disturbance.

It is well known that based on the error equation (33) a wide class of robust MRAC schemes was developed involving e.g. the use of small feedback around the "pure" integrator in the adaptive law, referred to as the σ-modification. The principal results are found in Ioannou and Sun (1996). Such robust MRAC schemes ensure closed-loop stability, and robustness with respect to the plant uncertainties $\Delta_m(s)$, and an external disturbance d by using various standard robust adaptive laws, see Ioannou and Sun (1996, Theorem 9.3.2). So, in view that in the tube MRAC case both stability and optimization can be resolved independently, the proposed reference model forming procedure can be an useful addition also and to an existing robust MRAC theory.

Remark 8. Note that in case **(ii)** singularity cannot occur in (4), and hence it is not required to use projection to update the scalar component $\theta_c(t)$ of the vector gain $\theta(t)$.

5. Concluding remarks

By using the concept of on-line goal adaptation, we present a new approach to MRAC of uncertain linear plants. The developed controllers not only guarantee closed loop stability, asymptotically tracking and robustness properties but enables the alleviation of the control cost with respect to some criterion. For this we formulate an additional optimization problem into the MRAC framework to find the newly defined correction control component at each instant of time. The technical viewpoint taken in our approach is that the regressor vector and basic error equation can be suitably modified into a form amenable for design and analysis using all earlier methods from which adaptive control laws can be directly obtained. A nice feature of the design approach is that the additional requirements are explicitly specified in the problem formulation. This is significant in order to succeed in designing practical controllers for applications with stringent requirements. The proposed approach provides a convenient intuitive interpretation of the design problem, while retaining the fundamental ideas on which model reference adaptive control is based. Hence this contribution is a significant complement to conventional MRAC in the contexts of control effort attenuation by on-line adaptation of the control goal. Applying the introduced concept to adaptive control problems leads to a new perspective and a host of new questions, only some of which are investigated in this paper. It is possible to extend the ideas presented here in several directions. These include the use of adaptive laws with and without normalization, combined direct and indirect approaches, Monopoli's augmented error, multiple models, backstepping, etc.

Acknowledgements

We thank the Associate Editor and the reviewers for their thoughtful and thorough reviews.

References

Åström, K. J., & Wittenmark, B. (1995). *Adaptive control*. New York: Addison-Wesley.
Horowitz, I. M. (1993). *Quantitative feedback design theory (QFT), vol. 1*. QFT Publications, Boulder, Co.
Ilchman, A., Ryan, E. P., & Townsend, P. (2007). Tracking with prescribed transient behavior for nonlinear systems of known relative degree. *SIAM Journal on Control and Optimization*, *46*(1), 210–230.
Ioannou, P. A., & Sun, J. (1996). *Robust adaptive control*. New Jersey: Prentice-Hall.
Joshi, S. M., Tao, G., & Patre, P. (2011). Direct adaptive control using an adaptive reference model. *International Journal of Control*, *84*(1), 180–196.
Krstić, M., Kanellakopoulos, I., & Kokotović, P. (1995). *Nonlinear and adaptive control design*. New York: John Wiley.
Mirkin, B. M. (2001). Decomposition-coordination optimization of the dynamic systems with adaptation of the criterion. *Automation and Remote Control*, *62*(7), 1155–1164.
Mirkin, B., & Gutman, P.-O. (2010). Lyapunov-based adaptive output-feedback control of mimo nonlinear plants with unknown, time-varying state delays. In *9-th IFAC workshop on time delay systems*. Prague, Czech Republic, June 7–9.
Mirkin, B. M., & Gutman, P.-O. (2010). Robust adaptive output-feedback tracking for a class of nonlinear time-delayed plants. *IEEE Transaction on Automatic Control*, *55*(10), 2418–2424.
Mirkin, B., Gutman, P.-O., & Shtessel, Y. (2012). Coordinated decentralized sliding mode MRAC with control cost optimization for a class of nonlinear systems. *Journal of the Franklin Institute*, *349*, 1364–1379.
Mirkin, B., Gutman, P.-O., & Sjöberg, J. (2011). Output-feedback MRAC with reference model tolerance of nonlinearly perturbed delayed plants. In *18th IFAC world congress* (pp. 6751–6756). Milan, Italy, August 22–September 2, 2011.
Mirkin, B. M., & Mirkin, E. L. (1999). Adaptive decentralized control with the interval model. *Vestnik MUK*, *5*(1), 3–10 (in Russian).
Narendra, K. S., & Annaswamy, A. M. (1989). *Stable adaptive systems*. New York: Prentice-Hall.
Sang, Q., & Tao, G. (2010). Adaptive control of piecewise linear systems: the state tracking case. In *Proc. American control conference* (pp. 4040–4045). Marriott Waterfront, Baltimore, USA, June 30–July 02.
Sastry, S., & Bodson, M. (1989). *Adaptive control. stability, convergence, and robustness*. Englewood Cliffs, New Jersey: Prentice-Hall.
Tao, G. (2003). *Adaptive control design and analysis*. New York: John Wiley.

Boris Mirkin received his Electrical Engineer degree from the Frunze Polytechnic Institute, Frunze, USSR, his Ph.D. (candidate of sciences) in automatic control from the Institute of Automation, Academy of Sciences of the Kirghiz SSR, Frunze, USSR, and his Dr.Sc. degree in automatic control from the Institute of Control Sciences of the USSR Academy of Sciences, Moscow in 1965, 1972 and 1988, respectively. He received the title of Professor from the Highest Academic Commission of the USSR, Moscow in 1991. From 1966 to 2001 he was Researcher, Head of Laboratory and Deputy Director at the Institute of Automation, Academy of Sciences of the Kirghiz SSR (now the Kyrgyz Republic). From 1995 to 2001 he held the positions of Professor at Kyrgyz-Russian (Slavic) and International Universities of the Kyrgyz Republic. In 2001, he joined the Faculty of Civil Engineering at the Technion, Israel Institute of Technology, Haifa, Israel, where he is currently a Scientific Advisor. He has published more than 120 technical papers, 2 books, and several book chapters. He has been the leader of numerous scientific projects for government and industry. His current area of interest includes robust adaptive control, decentralized adaptive control and time-delay systems.

Per-Olof Gutman was born in Hoganas, Sweden on 21 May 1949. He received his Civ.-Ing. degree in Engineering Physics (1973), his Ph.D. in Automatic Control (1982) and the title of Docent in Automatic Control (1988), all from the Lund Institute of Technology, Lund, Sweden. He received his MSE degree from the University of California, Los Angeles in 1977 as a Fulbright grant recipient. From 1973 to 1975 he taught mathematics in Tanzania. In 1983–84 he held a post-doctoral position at the Faculty of Electrical Engineering, Technion — Israel Institute of Technology, Haifa, Israel. From 1984–1990 he was a scientist with the Control Systems Section, El-Op Electro-Optics Industries Ltd, Rehovot, Israel. From 1989 he is with the Technion — Israel Institute of Technology, holding, since 2007 the position of Professor at the Faculty of Civil and Environmental Engineering. Gutman spent most summers from 1990–2008 as a visiting professor at the Division of Optimization and Systems Theory, Department of Mathematics, Royal Institute of Technology, Stockholm, Sweden. During his sabbaticals, Gutman was a visiting professor with Laboratoire d'Automatique, ENSIEG, Grenoble, France in 1995–96; was a visiting professor with Universita del Sannio, Benevento, Italy, in 2005–2006; and a researcher with the Commissariat a l'energie atomique, Interactive Robotics Laboratory, Fontenay-aux-Roses, France. His research interests include robust and adaptive control, control of complex non-linear systems such as paper machines and other electro-mechanical systems, computer aided design, off-road vehicle control and other control applications in agriculture, such as greenhouse modelling and control, water supply systems control, and traffic control. Gutman has (co-)authored 76 papers for reviewed international journals, has contributed to 6 books, and co-authored Qsyn — The Toolbox for Robust Control Systems Design for use with Matlab. He has supervised 30 graduate and post-doctoral students. He served as an associate editor of Automatica, and as an EU evaluator of research proposals.