ECE 528 **Math background problems** **Spring 2024**

These are optional additional exercises on background material. You don't need to submit them, but you can discuss them with me (and also with other students). Solving these may help you with the second homework and with the upcoming midterm.

1. a) Construct a sequence of continuous functions $\{f_n\}$ on $[0,1]$ which converges to the identically 0 function pointwise everywhere on $[0,1]$ (i.e., $\lim_{n\to\infty} f_n(x) = 0$ for all $x \in [0,1]$) but not uniformly in the sense of the L_∞ norm, i.e., it is *not* true that $\max_{x\in[0,1]} |f_n(x)| \to 0$ as $n \to \infty$.

b) As a small extra challenge, make your sequence $\{f_n\}$ uniformly bounded, i.e., $|f_n(x)| \leq M$ for all $x \in [0,1]$ and all n, where $M > 0$ is a fixed number.

c) If the sequence $\{f_n\}$ is monotone, i.e., $f_n(x) \leq f_{n+1}(x)$ for all x and all n, then pointwise everywhere convergence *does* imply uniform convergence. Try to prove this, but if you can't, look it up—it's a special case of Dini's Theorem.

2. True or false: the product of two locally Lipschitz functions is locally Lipschitz; the product of two globally Lipschitz functions is globally Lipschitz.

3. Prove that a sequence of ℓ_1 sequences of norm 1 (i.e., infinite probability vectors) converges to another ℓ_1 sequence of norm 1 (probability vector) uniformly in the sense of the ℓ_1 norm if and only if it converges to it element-wise.

Note that the statement becomes false if we remove the assumption that the limit is a probability vector. Indeed, the sequence (10000...), (010000...), (001000...) converges to (00000...) pointwise but not uniformly in the sense of the ℓ_1 norm.