Time and place. As announced earlier, the midterm exam will be held on Thursday, Oct 27, in class (2:00–3:15pm). There will be no conflict/make-up exam given at any other time.

Topics covered. The exam will cover all the material discussed in class up to Tuesday, Oct 18 (I will say in class where the exact cut-off point will be). The specific topics are:

- Linear algebra: its knowledge will not be directly tested on the exam but it will be assumed
- Linear state-space equations and their solutions
- Stability: general definitions, Lyapunov's second (direct) and first (indirect) methods, stability of LTI systems via eigenvalues and via the Lyapunov equation
- Controllability and observability (both for LTV and LTI systems)
- Transfer functions and their state-space realizations

What to bring. The exam is closed-book, closed-notes. You may bring one double-sided sheet of notes.

Tips for preparing. Make sure to follow up on all lecture material, readings, and homework problems and solutions. Compared to an average homework, the exam will be shorter and will focus on your knowledge of concepts, not your ability to do calculations.

Practice exam. See next page for a practice midterm exam. Solutions to it will be posted on the class website.

Office hours. I will hold special office hours the week of the exam, time to be announced. The TA will also hold regular TA office hours that week, to which you can bring any questions about past homeworks that you want to clear up.

1. A control system $\dot{x} = Ax + Bu$, $y = Cx$ transforms, under a change of basis $x = P\bar{x}$ in the state space, to a control system $\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u$, $y = \bar{C}\bar{x}$ (note that the input u and output y stay the same, only the state x changes).

a) Derive the formulas for the new matrices $\bar{A}, \bar{B}, \bar{C}$ in terms of the original matrices A, B, C.

b) Verify that the new system has the same transfer function (or transfer matrix) as the original one.

2. Consider the system

$$\dot{x} = \begin{pmatrix} a & 1 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$$

where $x \in \mathbb{R}^2$, $u \in \mathbb{R}$, and $a \in \mathbb{R}$ is an unknown parameter. Suppose this system is known to have the following two properties:

(i) If we turn off the control (set $u(t) = 0$ for all t), the solutions asymptotically converge to the origin from all initial states.

(ii) From every initial state it is possible to reach the origin *in finite time* using some control.

Based on this information, determine all possible values of the parameter a.

3. In class we defined the *unobservable subspace* of an LTV system $\dot{x} = A(t)x$, $y = C(t)x$ to be the nullspace (kernel) of the observability Gramian which is defined to be the matrix

$$M = \int_{t_0}^{t_1} \Phi^T(t, t_0) C^T(t) C(t) \Phi(t, t_0) dt$$

where Φ is the state transition matrix for $A(t)$ and t_0, t_1 are given times.

a) Prove that for an LTI system $\dot{x} = Ax$, $y = Cx$ the unobservable subspace equals the nullspace of the observability matrix $\mathcal{O}(A, C)$.

b) Briefly explain the meaning of the unobservable subspace in the context of recovering x from y (again for the LTI case).