Time and place. As announced earlier, the midterm exam will be held on Thursday, Oct 26, in class (2:00–3:20pm). There will be no conflict/make-up exam given at any other time.

Topics covered. The exam will cover all the material discussed in class up to the lecture on Thursday, Oct 19 (I will say during that lecture where the exact cut-off point will be). The specific topics are:

- Linear algebra: its knowledge will not be directly tested on the exam but it will be assumed
- Linear state-space equations and their solutions
- Stability: general definitions, Lyapunov’s second (direct) and first (indirect) methods, stability of LTI systems via eigenvalues and via the Lyapunov equation
- Controllability and observability (both for LTV and LTI systems)
- Transfer functions and their state-space realizations

What to bring. The exam is closed-book, closed-notes. You may bring one double-sided sheet of notes. A calculator will not be necessary or helpful.

Tips for preparing. Make sure to follow up on all lecture material, readings, and homework problems and solutions. Compared to an average homework, the exam will be shorter and will focus on your knowledge of concepts, not your ability to do calculations.

Practice exam. See next page for a practice midterm exam. Solutions to it will be posted on the class website.

Office hours. I will hold special office hours the week of the exam, time to be announced. The TA will also hold regular TA office hours that week, to which you can bring any questions about past homeworks that you want to clear up.
1. Compute the state transition matrix $\Phi(t, t_0)$ of the linear time-varying system $\dot{x} = A(t)x$ with

$$A(t) = \begin{pmatrix} 1 & e^t + \frac{1}{2}t^2 \\ 0 & -t \end{pmatrix}$$

2. Consider the system

$$\begin{align*}
\dot{x}_1 &= x_1 \\
\dot{x}_2 &= x_2 \\
y &= x_1 - x_2
\end{align*}$$

a) Using the theory discussed in class, identify the set of initial conditions that cannot be distinguished from the zero initial condition on the basis of output observations.

b) Support your answer from part a) by drawing a picture. Argue with the help of that picture why the initial conditions you identified in part a)—and only these ones—are indistinguishable from 0.

3. Consider the following claim: the LTI system $\dot{x} = Ax$ is stable (in the sense of Lyapunov) if and only if for every symmetric matrix $Q$ with at least one eigenvalue at 0 but no negative eigenvalues, there exists a symmetric positive definite matrix $P$ satisfying the Lyapunov equation $PA + A^TP = -Q$. (All matrices are $n \times n$ for the same given $n$.)

a) Is the “if” part of the above claim correct?

b) Is the “only if” part of the above claim correct?