Hybrid control of V/STOL aircrafts - Final Project for ECE497DL

Kwangsoo Kim

May 14, 2001

Abstract

A V/STOL aircraft is highly nonlinear and slightly nonminimum phase due to the input coupling terms. However, the V/STOL aircraft is appropriate for the hybrid modeling according to the operation mode such as CTOL mode, VTOL mode, and transition mode. In this report, the modeling of the V/STOL is illustrated and nonminimum phase characteristics of the system is explained using the zero dynamics. Also, controllers in each mode will be designed using the approximate linearization method published in some literatures. The switching stability will be shown using the common Lyapunov function.

1 Introduction

Unlike the conventional take-off and landing(CTOL) aircrafts, the vertical take-off and landing (VTOL) has obvious advantage that they don't need to fixed runway for take-off and landing, which provides a lot of flexibility to military tactics. So researches on the vehicles to leave the ground vertically has a long history even before fixed-wing aviation became widely established via CTOL [3]. The V/STOL aircrafts can be classified according to the powered lift principle: vectored thrust, separate lift engines, and air flow-switchers [3]. In this report, the vectored thrusted V/STOL aircraft will be dealt with.

However, despite of the advantageous flexibility, V/STOL has a characteristics such as [5, 10]

1. **Poor inherent stability:** The aircraft is longitudinally unstable at low speed. It has a divergent response in roll which means the pilot must ensure that he doesn't put on too much bank angle. In yaw there

is very low directional stability, and high yaw rates at low speeds are potentially lethal.

2. **Poor decoupling:** The pilot's inceptors do not command directly what the pilot wishes to control; for example, a change in nozzle lever setting alters both vertical and horizontal accelerations.

   The pilot must manually control both the thrust and the nozzle angle with the same hand, stepping the nozzle through predefined angle sequences while adjusting the throttle accordingly.

3. **Appropriate for a hybrid modeling:** The dynamic behavior of the aircraft can be abstracted into three modes: a conventional take-off and landing (CTOL) mode, vertical take-off and landing (VTOL) mode, and transition mode.

4. **Nonminimum phase property:** The analysis of the aircraft dynamics shows that nonminimum phase characteristics.

Like in the [9, 10], the V/STOL aircraft is modeled into three modes according to different operation and nozzle angle. Such a scheme is appropriate for hybrid modeling. So the V/STOL model can be viewed as a hybrid system such as

$$\dot{x} = f_\sigma(x, u)$$

where $\sigma \in P = \{1, 2, 3\}$ indicating CTOL mode ($\sigma = 1$), VTOL mode ($\sigma = 2$), and transition mode ($\sigma = 3$). The model in each mode will be obtained using free body diagrams and nonminimum phase property will be shown by analyzing the zero dynamics. Due to the difficulty in quantitative analysis, the unstable phenomenon of the zero dynamics will be shown through the phase portrait of the system for the various initial conditions.

With such a hybrid model, the controller for guaranteeing an approximate tracking in each mode will be designed using the approximate linearization method. Analysis and simulation with the controller will show that the system is stable for all σ. After checking about the stability in each mode, the switching stability will be discussed. A fairly well-known fact about the switching stability is that a family of systems $\dot{x} = f_p(x), p \in P$ is stable if there exists a common Lyapunov function $V(x, t)$ for all $x \neq 0$ and all $p \in P$ [7]. Choosing an appropriate common Lyapunov function for this hybrid system will show that the overall stability.

2 Modeling

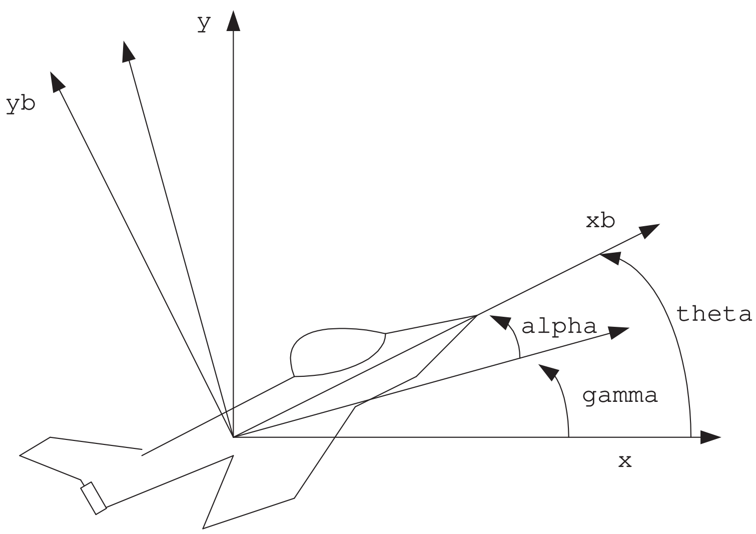
The horizontal and vertical axes are respectively the (x, z) axes and the pitch angle θ is the angle between the aircraft body axis, x_b, and the x axis. The flight path angle γ, the angle of attach α, and the ground speed V, are defined as $\gamma = \tan^{-1}(\frac{\dot{z}}{\dot{x}}), \alpha = \theta - \gamma$, and $V = \sqrt{\dot{x}^2 + \dot{z}^2}$, respectively. The aerodynamic equations for lift(L) is given by

$$\begin{aligned} L &= c_L \cdot (\text{wing area}) \cdot \frac{1}{2}\rho V^2 \\ &= a_L V^2 (1 + c\alpha) \end{aligned} \tag{1}$$

where a_L is dimensionless *lift coefficient*, ρ is the density of air, and c_L, c are positive integers. The drag(D) of an aircraft is given by

$$\begin{aligned} D &= \frac{1}{15} a_L (1 + b(1 + c\alpha)^2) V^2 \\ &= a_D V^2 (1 + b(1 + c\alpha)^2) \end{aligned} \tag{2}$$

where a_D is dimensionless *drag coefficient* and b is a positive integer.



2.1 CTOL mode

In this mode, the nozzle are at 0° relative to the horizontal body axis. The free body diagram in this mode is shown in Figure 2.1. According to this free body diagram, the dynamic equations are given by:

$$M \begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} = R(\theta) \left(R^T(\alpha) \begin{bmatrix} -D \\ L \end{bmatrix} + \begin{bmatrix} T \\ -\epsilon u_2 \end{bmatrix} \right) - \begin{bmatrix} 0 \\ Mg \end{bmatrix} \tag{3}$$

where $R(\alpha)$ and $R(\theta)$ are standard rotation matrices such as

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

and ϵ is a small positive constant. The values of coefficients in (3) are adopted from [10] as follows: $M = 16280$ lb, $J = 32000$ slug-ft^2, $b = 0.002, c = 39.97, a_L = 0.78$, and $a_D = 2.62$.

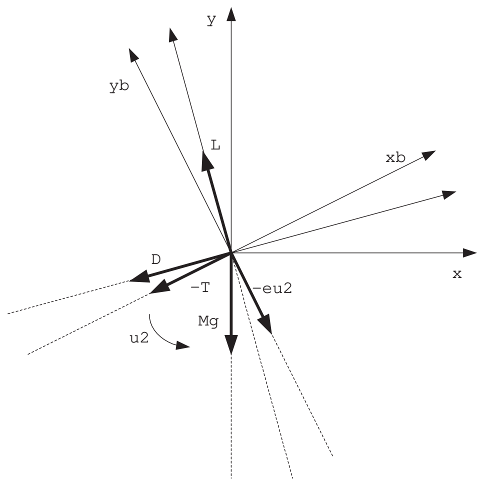


Figure 1: Free body diagram in CTOL mode

Using the definition of ground speed, equation for the linear acceleration may be derived as follows:

$$\begin{aligned} V &= \sqrt{\dot{x}^2 + \dot{z}^2} \\ &= \dot{x}\sqrt{1 + \tan^2\gamma} \\ &= \dot{x} \cdot \sec\gamma \end{aligned}$$

So,

$$\dot{V} = \frac{\ddot{x}\cos\gamma + \dot{x}\sin\gamma \cdot \dot{\gamma}}{\cos^2\gamma}$$

However,

$$\dot{\gamma} = \frac{\left(\frac{\dot{z}}{\dot{x}}\right)'}{1 + \left(\frac{\dot{z}}{\dot{x}}\right)^2} = \cos^2\gamma \cdot \left(\frac{\dot{z}}{\dot{x}}\right)'$$

Hence,

$$\begin{aligned}
\dot{V} &= \frac{1}{\cos^2\gamma}\left(\ddot{x}\cos\gamma + \dot{x}\sin\gamma\frac{\ddot{z}\dot{x} - \dot{z}\ddot{x}}{\dot{x}^2}\cdot\cos^2\gamma\right) \\
&= \frac{1}{\cos\gamma}\ddot{x} + \sin\gamma\frac{\ddot{z}\dot{x} - \dot{z}\ddot{x}}{\dot{x}} \\
&= \frac{1}{\cos\gamma}\ddot{x} + \sin\gamma\cdot\ddot{z} - \frac{\sin^2\gamma}{\cos\gamma}\ddot{x} \\
&= \cos\gamma\ddot{x} + \sin\gamma\ddot{z} \qquad (4)
\end{aligned}$$

Using the (3) and (4) under the assumption that $\sin\alpha \approx \alpha, \cos\alpha \approx 1$, and $\epsilon/M \approx 0$ since α is typically small in this mode, the equation for the linear acceleration can be obtained like as:

$$\dot{V} = -\frac{D}{M} - g\sin\gamma + \frac{T}{M} + \frac{L}{M}\alpha \qquad (5)$$

2.2 VTOL mode

In this mode, the nozzle are at 90° relative to the horizontal body axis. The free body diagram in this mode is shown in Figure 2.2. In this mode, the ground speed is low enough that we can assume the lift and drag forces are negligible. Refer to the (1) and 2).

The dynamic equations are given by:

$$M\begin{bmatrix}\ddot{x}\\ \ddot{z}\end{bmatrix} = R(\theta)\begin{bmatrix}0\\ T - \epsilon u_2\end{bmatrix} - \begin{bmatrix}0\\ Mg\end{bmatrix} \qquad (6)$$

The equation for the linear acceleration may be derived from (4), (6) and the simple mathematical fact that $\sin(\gamma - \theta) = \sin\gamma\cos\theta - \cos\gamma\sin\theta$ with the assumption that $\epsilon/M \approx 0$:

$$\dot{V} = -g\sin\gamma - \frac{T}{M}\sin\alpha \qquad (7)$$

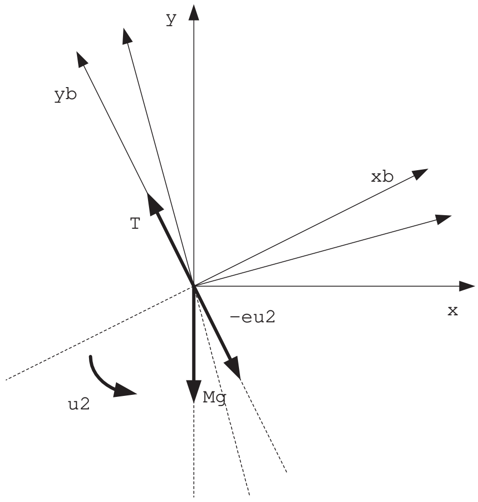


Figure 2: Free body diagram for VTOL mode

2.3 Transition mode

In this mode, the nozzle is between 0° and 90°. Therefore, the additional states which model the dynamics of the nozzles are necessary. So the new state δ is introduced here for the nozzle angle. In this mode, the free diagram can be modified shown as Figure 2.3.

According to the free body diagram, the longitudinal dynamics equations are:

$$M \begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} = R(\theta)\left(R^T(\alpha)\begin{bmatrix} -D \\ L \end{bmatrix} + \begin{bmatrix} T\cos\delta \\ T\sin\delta - \epsilon u_2 \end{bmatrix}\right) - \begin{bmatrix} 0 \\ Mg \end{bmatrix} \tag{8}$$

This leads to the longitudinal axis dynamics again using the simple mathematical formulae $\cos(\gamma-\theta) = \cos\gamma\cos\theta + \sin\gamma\sin\theta$ and $\sin(\gamma-\theta) = \sin\gamma\cos\theta - \cos\gamma\sin\theta$.

$$\dot{V} = -\frac{D}{M} - g\sin\gamma + \frac{T}{M}\cos(\alpha+\delta) \tag{9}$$

2.4 Zero dynamics

One way of analyzing the internal stability is to look at the zero dynamics of the system. When x and z are chosen as outputs and T and $u_2 = J\ddot{\theta}$ as inputs, exact input-output linearization of the system is considered using

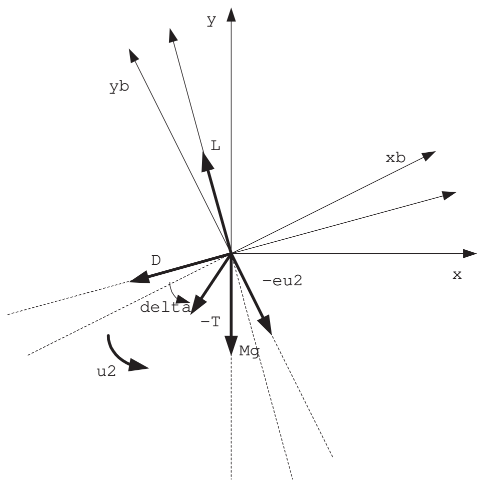


Figure 3: Free body diagram for transitional mode

the state feedback law of the form [6]

$$u = \alpha(x) + \beta(x)v,$$

such that, for some r (which is called the relative degree of the system),

$$y^{(r)} = v$$

where v is the new input and x is the entire of the system and y denotes the outputs.

In the CTOL mode, according to (3), we can find the state feedback law which linearizes the system as follows:

$$\begin{bmatrix} T \\ \epsilon u_2 \end{bmatrix} = -R^T(\alpha) \begin{bmatrix} -D \\ L \end{bmatrix} + R^T(\theta) \begin{bmatrix} Mv_1 \\ Mv_2 + Mg \end{bmatrix} \tag{10}$$

The resulting system is

$$\begin{aligned} \ddot{x} &= v_1, \\ \ddot{z} &= v_2, \\ \ddot{\theta} &= \frac{1}{J\epsilon}(D\sin\alpha + L\cos\alpha + Mv_1\sin\theta - Mv_2\cos\theta - Mg\cos\theta) \end{aligned} \tag{11}$$

The zero dynamics of a nonlinear system are the internal dynamics of the system subject to the constraint that the outputs are set to zero for all time. So the zero dynamics can be obtained by setting $v_1 = v_2 = 0$. Note that the system is said to be *minimum phase* if the zero dynamics has an asymptotically stable equilibrium point in the domain of interest and *nonminimum phase* otherwise [6]. Hence, in the CTOL mode, the zero dynamics of the system is

$$\ddot{\theta} = \frac{1}{J\epsilon}(D \sin\alpha + L\cos\alpha - Mg\cos\theta)$$

Since putting $\ddot{x} = \ddot{z} = 0$ corresponds to the aircraft flying at a constant altitude and a constant horizontal velocity, we can assume that $\alpha \approx \theta$ so that the zero dynamics in the CTOL mode can be written as

$$\ddot{\theta} = \frac{1}{J\epsilon}(D \sin\theta + L\cos\theta - Mg\cos\theta) \tag{12}$$

Inserting (1) and (2) into (12), we obtain as follows:

$$\ddot{\theta} = \frac{1}{J\epsilon}\left\{a_D V^2(1 + b(1 + c\theta)^2)\sin\theta + a_L V^2(1 + c\theta)\cos\theta - Mg\cos\theta\right\} \tag{13}$$

The quantitative analysis of (13) does not look simple and clear. So qualitative analysis using simulation is attempted. The behavior of the zero dynamics are illustrated with a simulation. Figure 4 shows a phase portrait of the zero dynamics where the ground velocity, V, was chosen as $V = 0.85/\sqrt{a_L}$ to guarantee that the saddle point of the zero dynamics lies at about $\theta = 0.06$ radians. [11].

From this phase portrait, it can be known that all the initial points result in unbounded solutions. For example, if the initial condition is $(\theta, \dot{\theta}) = (-2.0, 0)$, the phase portrait is shown in Figure 5. The solutions for this initial condition behaves like a unstable focus.

So, the zero dynamics is unstable, which means that the system is non-minimum phase system.

In the Transition mode, using the dynamic feedback as follows:

$$\begin{bmatrix} T\cos\delta \\ T\sin\delta - \epsilon u_2 \end{bmatrix} = -R^T(\alpha)\begin{bmatrix} -D \\ L \end{bmatrix} + R^T(\theta)\begin{bmatrix} Mv_1 \\ Mv_2 + Mg \end{bmatrix} \tag{14}$$

the zero dynamics in the transition mode can be obtained:

$$\ddot{\theta} = \frac{1}{J\epsilon}\{D(\sin\theta + \cos\theta\tan\delta) - L(\cos\theta - \sin\theta\tan\delta) + Mg(-cos\theta + \sin\theta\tan\delta)\} \tag{15}$$

Here, the assumption that $\alpha \approx \theta$ was used. With the fixed value $\delta = \frac{\pi}{3}$, the phase portrait of the system is shown in Figure 6.

Alike the CTOL mode, it can be deduced that the zero dynamics is unstable so that the original system is an nonminimum phase system.

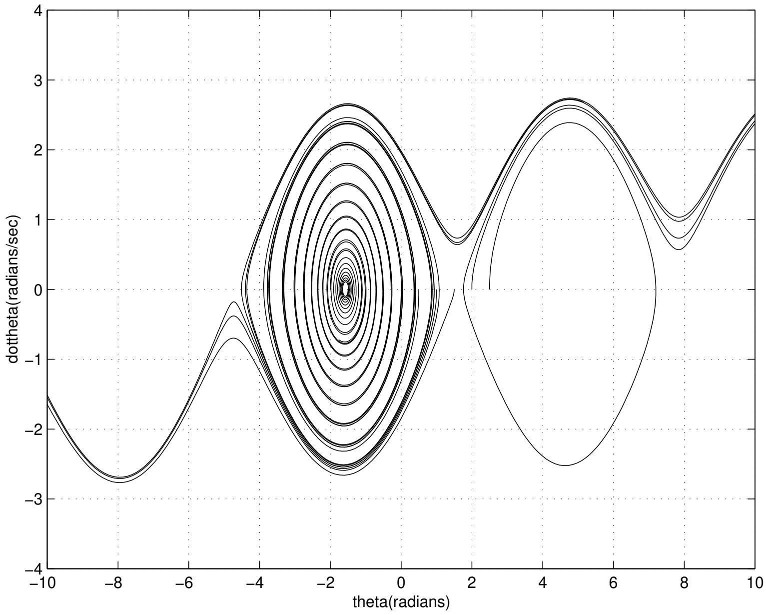


Figure 4: Phase portrait of zero dynamics

3 Controller design in each mode

There are some approaches to output tracking used on systems with unstable zero dynamics: flat output method which relies on choosing outputs that makes the system "flat" [8], Devasia and Paden's inversion-based output tracking method which is based on the solution of the stable inversion problem [2], the Hauser's approximate linearization techniques [4], and Benvenuti's approximate linearization method [1]. The [11] explained each method briefly and compared its characteristics. In this report, the Hauser's approximate linearization method are used to design a controller for nonminimum phase system.

3.1 Approximate linearization method

Most materials in this section are adopted from [4].

Consider the class of nonlinear systems of the form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{16}$$

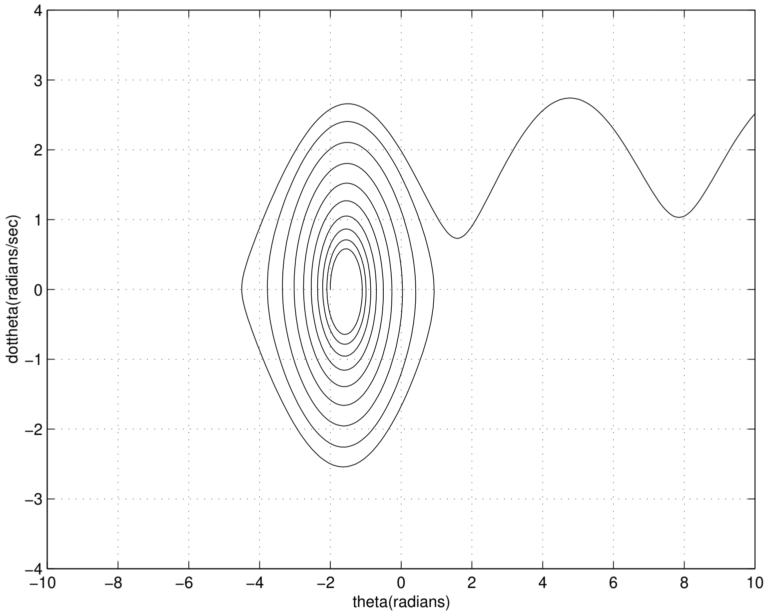


Figure 5: Phase portrait for the initial condition $(\theta, \dot{\theta}) = (-2.0, 0)$

where $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \to \mathbb{R}^n$, the columns of $g : \mathbb{R}^n \to \mathbb{R}^n$ are smooth vector fields ,and $h : \mathbb{R}^n \to \mathbb{R}$ is a smooth function with $h(0) = 0$. We consider here single input and single output case for brevity, and so assume that $u \in \mathbb{R}, y \in \mathbb{R}$.

Suppose that $L_g h(x) = \epsilon\psi(x)$ for some scalar function $\psi(x)$ with $\epsilon > 0$ small, where $L_g h(x)$ is the Lie derivative of $h(\cdot)$ along $g(\cdot)$. Define two systems in normal form using the following two sets of local diffeomorphisms of $x \in \mathbb{R}^n$,

$$(\xi^T, \eta^T)^T = (\xi_1 = h(x), \eta_1(x), \ldots, \eta_{n-1}(x))^T,$$

and

$$(\tilde{\xi}^T, \tilde{\eta}^T)^T = (\tilde{\xi}_1 := h(x), \tilde{\xi}_2(x) = L_f h(x), \tilde{\eta}(x), \ldots, \tilde{\eta}_{n-2}(x))^T,$$

with

$$\frac{\partial \eta_i}{\partial x} g(x) = 0, \qquad i = 1, \ldots, n-1,$$

and

$$\frac{\partial \tilde{\eta}_i}{\partial x} g(x) = 0, \qquad i = 1, \ldots, n-2.$$

- (true system)

$$
\begin{aligned}
\dot{\xi}_1 &= L_f h(x) + L_g h(x) u \\
\dot{\eta} &= q(\xi, \eta)
\end{aligned}
\tag{17}
$$

- (approximate system)

$$
\begin{aligned}
\dot{\tilde{\xi}}_1 &= \tilde{\xi}_2 \\
\dot{\tilde{\xi}}_2 &= L_f^2 h(x) + L_g L_f h(x) u \\
\dot{\tilde{\eta}} &= \tilde{q}(\tilde{\xi}, \tilde{\eta})
\end{aligned}
\tag{18}
$$

In [4], the system (16) is called *slightly nonminimum phase* if the true system (17) is nonminimum phase but the approximate system (18) (with $\epsilon = 0$) is minimum phase.

The strategy is to use the input-output linearizing control law for the approximate system (18) such as

$$
u^* = \frac{1}{L_g L_f h(x)}(-L_f^2 h(x) + v), \tag{19}
$$

where v is chosen depending on the control task. If, for example, v is chose as

$$
v = \ddot{y}_d + \alpha_1(\dot{y}_d - \tilde{\xi}_2) + \alpha_0(y_d - \tilde{\xi}_1)
$$

with the definition of errors,

$$
e_1 = \tilde{\xi}_1 - y_d, \; e_2 = \tilde{\xi}_2 - \dot{y}_d
$$

the error dynamics between the true system and the approximate system is given by

$$
\begin{aligned}
\dot{e}_1 &= e_2 + \epsilon\psi(x) u^*(x) \\
\dot{e}_2 &= -\alpha_1 e_2 - \alpha_0 e_1 \\
\dot{\tilde{\eta}} &= \tilde{q}(\tilde{\xi}, \tilde{\eta})
\end{aligned}
\tag{20}
$$

When this error system can be designed to be stable and the zero dynamics of the approximate system is stable exponentially, the overall stability of the system and approximate tracking are guaranteed.

Theorem 1 (Hauer-Sastry-Meyer [4]) *Suppose that the zero dynamics of the approximate system are locally exponentially stable and the function $\psi(x)u^*(x)$ is locally Lipschitz continuous. Then, for ϵ sufficiently small and for the desired trajectories with small sufficiently values and derivatives, the states of the true system will be bounded and the tracking error*

$$
|e_1| = |\tilde{\xi}_1 - y_d| \leq k\epsilon,
$$

3.2 controller design in CTOL mode

As explained in the previous section, the input coupling term, ϵ, is set to zero in the approximate linearization method. Then, the equation in the CTOL mode becomes:

$$M \begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} = R(\theta) \left(R^T(\alpha) \begin{bmatrix} -D \\ L \end{bmatrix} + \begin{bmatrix} T \\ 0 \end{bmatrix} \right) - \begin{bmatrix} 0 \\ Mg \end{bmatrix} \tag{21}$$

4 Future works

In this report, the simple model of V/STOL was obtained using the free body diagrams in each mode and phase portrait analysis of zero dynamics showed that the V/STOL is nonminimum phase. Among the various methods for nonlinear nonminimum phase system, the approximate linearization method of Hauser's is summarized briefly. However, the specific control law using this method and simulation are not completed. So the obtaining control law for guaranteeing the output tracking as well as stability should be followed in the future. Also the switching stability analysis is needed.

5 appendix: m-file for phase portrait

```
clear all;

aD = 2.62;
aL = 0.78;
Vsquare = (0.85^2)/0.78;
b = 0.002;
c =39.97;
t = 0.05;
inv_Je = 1/(32000 * 0.3);
Mg = 16280;

for i=1:10;
   x(1) = (i-5)*0.5;
   for k=1:4000;
      D = aD*Vsquare*(1+b*(1+c*x(k))^2);
      L = aL*Vsquare*(1+c*x(k));
      ddot_x(k) = inv_Je * (D*sin(x(k)) + (L-Mg)*cos(x(k)));
      dot_x(k+1) = t *ddot_x(k) + dot_x(k);
```

```
      x(k+1) = t*dot_x(k)+x(k);
   end
plot(x, dot_x);
hold on;
clear x; clear dot_x; clear ddot_x;
end

grid;
axis([-10 10 -4 4]); xlabel('theta(radians)');
ylabel('dottheta(radians/sec)');
```

References

[1] L. Benvenuti, M. D. Di Benedetto, and J. W. Grizzle. Approximate output tracking for nonlinear nonminimum phase systems with an application to flight control. *Journal of Nonlinear and Robust control*, 4:397–414, 1993.

[2] S. Devasia and B. Paden. Exact output tracking for nonlinear time-varying systems. In *Conference on Decision and Control*, pages 2346–2355, 1994.

[3] J. W. Fozard. Tactical jet v/stol - its future in a ctol world. In *Aerospace Technology Conference and Exposition*, pages 33–57, Long Beach, CA, October 1986.

[4] John Hauser, Shankar Sastry, and George Meyer. Nonlinear control design for slightly nonminimum phase systems: Application to v/stol aircraft. *Automatica*, 28(4):665–679, 1992.

[5] R. A. Hyde. *H_∞ Aerospace Control Design: A VSTOL Flight Application.* Springer-Verlag, London, 1995.

[6] Hassan K. Khalil. *Nonlinear systems.* Prentice Hall, Upper Saddle River, NJ, 2nd edition, 1996.

[7] D. Liberzon and A. S. Morse. Basic problems in stability and design of switched systems. *IEEE Control Systems Magazine*, 19(5):59–70, 1999.

[8] P. Martin, S. Devasia, and B. Paden. A different look at output tracking: control of a vtol aircraft. In *Conference on Decision and Control*, pages 2376–2381, 1994.

[9] M. Oishi and C. Tomlin. Switched nonlinear control of a vstol aircraft. In *Conference on Decision and Control*, pages 2685–2690, Phoenix, AZ, December 1999.

[10] M. Oishi and C. Tomlin. Switching in nonminimum phase systems: applications to a vstol aircraft. In *American Control Conference*, pages 487–491, Chicag, IL, June 2000.

[11] Claire Tomlin, John Lygeros, Luca Benvenuti, and Shankar Sastry. Output tracking for a non-minimum phase dynamic ctol aircraft model. In *Conference on Decision and Control*, pages 1867–1872, New Orleans, LA, December 1995.

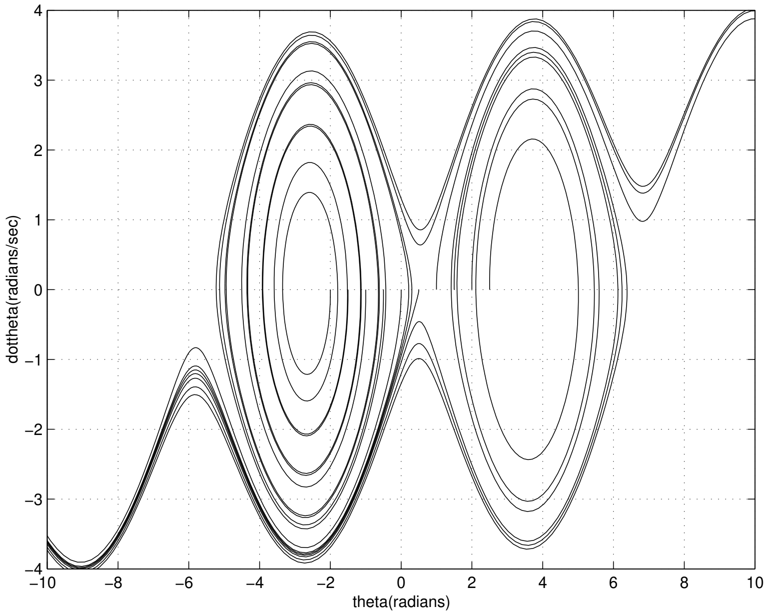


Figure 6: Phase portrait of the transition mode when $\delta = \pi/3$