Multiple Model Adaptive Control With Mixing

Matthew Kuipers and Petros Ioannou, *Fellow, IEEE*

***Abstract*—Despite the remarkable theoretical accomplishments and successful applications of adaptive control, the field is not sufficiently mature to solve challenging control problems where strict performance and robustness guarantees are required. Critical to the design of practical control systems for these challenging applications, and currently lacking in parameter estimation-based adaptive control schemes, is an approach that explicitly accounts for robust-performance and stability specifications. Towards this goal, this paper describes a robust adaptive control approach called *adaptive mixing control* that makes available the full suite of powerful design tools from LTI theory, e.g., mixed-μ synthesis. The stability and robustness properties of adaptive mixing control are analyzed. It is shown that the mean-square regulation error is of the order of the modeling error provided the unmodeled dynamics satisfy a norm-bound condition. And when the parameter estimate converges to its true value, which is guaranteed if a persistence of excitation condition is satisfied, the adaptive closed-loop system converges exponentially fast to a closed-loop system comprising the plant and some LTI controller that satisfies the control objective. A benchmark example is presented, which is used to compare the adaptive mixing controller with other adaptive schemes.**

***Index Terms*—Multiple model adaptive control, robust adaptive control.**

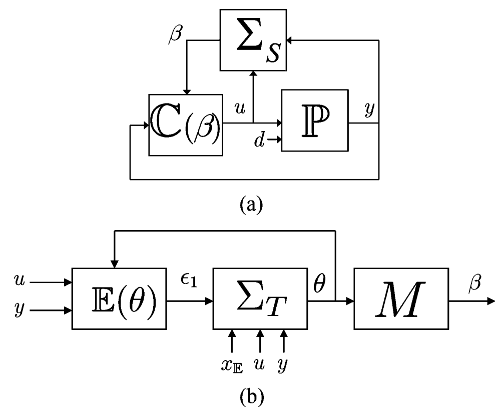

Fig. 1. Multiple model adaptive control architecture and adaptive mixing control supervisor (a) Multiple model adaptive control architecture: Based on observed data, the supervisor Σ_S selects/blends/mixes candidate controllers (b) Adaptive mixing control supervisor: The adaptive law Σ_T adjusts the estimate $\theta(t)$ to make the estimation error $\epsilon_1(t)$ small. The mixing signal $\beta(t)$ mixes the candidate controllers by considering the estimate as the true unknown parameter.

I. Introduction

When model uncertainties are sufficiently small, modern linear time invariant (LTI) control theories, e.g., $\mathcal{H}_\infty$ and μ-synthesis [1]–[3], ensure, when possible, satisfactory closed-loop objectives specified in meaningful engineering terms (frequency weights on the relevant transfer functions) are met. However, changes in operating conditions, failure or degradation of components, or unexpected changes in system dynamics may all violate the assumption of small uncertainty, particularly parametric uncertainty. The impact of such "large" uncertainty is that a single fixed LTI controller may no longer achieve satisfactory closed-loop behavior, let alone stability. What is needed is a controller that is able to monitor the plant dynamics in order to adjust its control law to compensate for such parametric uncertainty and other modeling errors [4, Sec. 1.3].

Adaptive control copes with large parametric uncertainty by tuning controller gains in response to estimated changes in the model. Since in conventional (robust) adaptive control [4], [5] the controller gains are calculated in real time based on the estimated plant model, the complicated relationship between plant parameters and $\mathcal{H}_\infty$ and μ-synthesis controller gains has precluded the use of conventional adaptive versions of these modern robust compensators.

By using candidate controllers designed off-line, the multiple model adaptive control (MMAC) architecture, shown in Fig. 1(a), avoids real-time controller synthesis issues and, therefore, provides an attractive framework for combining adaptive and modern robust tools. The MMAC architecture comprises two levels of control: (1) a low-level system $\mathbb{C}(\beta)$ called the multicontroller that is capable of generating finely-tuned candidate controls and (2) a high-level system Σ_S called the supervisor that influences the control u by adjusting the multicontroller, typically by selecting or weighting candidate controllers, based on processed plant input/output data.

The focus of this paper is the presentation and analysis of a novel MMAC architecture, *adaptive mixing control*, that "mixes" the candidate controllers in a continuous manner based on a robust adaptive law. The multicontroller is not only capable of generating any of the candidate control laws but also, by controller interpolation, a stable mix of candidate control laws. This mixing behavior allows the multicontroller to evolve from one controller to another in a continuous manner. Moreover, provided certain conditions on the plant input are met, the adaptive mixing controller converges exponentially fast to meet the control objective.

The supervisor, shown in Fig. 1(b), generates the mixing signal $\beta(t)$ by processing the estimate $\theta(t)$ of the unknown plant parameters $\theta(t)$ with a system called the mixer M that determines the level of participation of the candidate controllers. This determination is a manifestation of certainty equivalence: at every fixed $t \geq 0$, the candidate controllers that

Manuscript received May 06, 20098 December, 2009; revised November 10, 2009; accepted December 08, 2009. First published February 05, 2010; current version published July 30, 2010. This work was supported by the National Science Foundation under Grant 0510921. Recommended by Associate Editor A. Astolfi.

The authors are with the Department of Electrical Engineering, University of Southern California, Los Angeles, CA 90024 USA (e-mail: kuipers@usc.edu; ioannou@usc.edu).

Digital Object Identifier 10.1109/TAC.2010.2042345

were designed for $\theta^* = \theta(t)$ are mixed such that closed-loop objectives are met.

The MMAC concept is not new and has been around for quite some time. One recent approach is the so-called supervisory control [6]–[8], in which controller selection is made by continuously comparing in real time suitably defined norm-squared estimation errors, also referred to as performance signals, and the candidate controller associated with the smallest performance signal is placed in the loop according to an appropriate switching logic. Following the idea of supervisory control, logic-based switching and multiple models were combined with conventional adaptive control [9]–[11] with the objective of improving transient performance of conventional adaptive schemes. Also incorporating logic-based switching is the so-called unfalsified control approach [12], [13], which is a nonidentifier-based deterministic approach. The unfalsified control approach is a model free approach and differs from most other switching schemes. It relies on measured data to select the right controller. Even though the method guarantees convergence to a stabilizing robust controller, the simulation studies in [14], [15] report unacceptable transients. This is consistent with intuition: at the outset, measured input-output data may largely reflect initial conditions, resulting in the selection of a poorly performing controller for an extended period of time until measurement quality improves. Therefore the claim in [12], [13] that no knowledge of the plant model or its form is required is true for stability but not for performance unless further modifications are added as discussed in [15].

Switching-based schemes have a number of advantages. Switching in adaptive control was originally introduced as a method to overcome the loss of stabilizability in parameter estimation based adaptive control [16], [17]. Also, these schemes have the advantage of rapid adaptation to large, abrupt parameter changes. This is a desirable switching behavior. Switching, however, may exhibit undesirable behaviors that could negatively affect performance. Explained heuristically, if hysteresis is used and the true model is near the boundary of two candidate models, the supervisor may persistently select a controller that does not achieve desirable closed-loop behavior, despite observed data indicating an acceptable candidate controller is preferred. To encourage switching, the hysteresis constant may be reduced or replaced with a dwell-time logic, but at the increased risk of long-term intermittent switching between multiple controllers, resulting in transients from improper initialization of the new controller[1].

Another promising MMAC approach is based on the so-called robust MMAC (RMMAC) methodology that provides guidelines for designing both the candidate controller set (using mixed-μ synthesis tools) and the supervisor [18]–[20]. The RMMAC approach originated from the multiple model adaptive estimation/MMAC methods [21] of the 1970s, of which there have been numerous successful applications based on adaptations of these methods [22]–[24]. The RMMAC supervisor is based on a dynamic hypothesis testing scheme that generates for each candidate the posterior probability that its model is "closest" to the true plant. These probabilities are used to weight the candidate controller outputs, or, as done in the RMMAC/S variant, to switch into the loop the candidate controller associated with the highest posterior probability. Given accurate disturbance and noise models that satisfy the standard Kalman filter assumptions, simulations demonstrate rapid adaptation and superior performance compared to a nonadaptive mixed-μ compensator [20]. Acknowledged within the same reference, however, is that special care is needed to compensate for an inaccurate stochastic disturbance model. The RMMAC/XI architecture was proposed to handle a range of disturbance powers, at the cost of additional Kalman filters. Still, if the disturbance power is significantly outside the expected range, poor performance may occur. And although loss of stabilizability is not an issue (because there is no estimated model), it should be noted that no stability results have been published.

The immediate motivation of adaptive mixing control is to provide an adaptive control approach that is capable of incorporating the full suite of powerful LTI tools, while avoiding some of the performance issues associated with undesirable switching phenomena and an unknown or uncertain disturbance model, offering an alternative to the existing MMAC approaches for particular applications. The unique feature of adaptive mixing control with respect to existing MMAC approaches, including RMMAC, is that the intent is not to converge to one controller, but rather a stable mix of candidate controllers. This mixing behavior avoids switching and, in turn, some of its undesirable behaviors. Also, while an adaptive mixing control scheme's performance may be improved by incorporating *a priori* knowledge of the disturbance, the evaluations in Section VI and [25], where the latter focuses on a multiple estimator variant of the approach presented in this paper, demonstrate that adaptive mixing can achieve satisfactory performance despite significant perturbations in the disturbance power and bandwidth. Last, utilizing pre-computed controllers, adaptive mixing control avoids computational and existence issues of calculating controller gains when stabilizability of the estimated plant is lost.

Notation and Preliminaries

Suppose that A is a $m \times n$ matrix. The transpose of A is denoted by A^T. For a n-vector x, $|x|$ is the Euclidean norm $(x^T x)^{1/2}$ and the corresponding induced matrix norm of A is denoted as $\|A\|$. If $x(t)$ is a function of time, then the $\mathcal{L}_p$ norm of x is denoted as $\|x\|_p$ and the truncated $\mathcal{L}_{2\delta}$ norm is defined as $\|x_t\|_{2\delta} \triangleq \left(\int_0^t e^{-\delta(t-\tau)}|x(\tau)|^2 d\tau\right)^{1/2}$, where $\delta \geq 0$ is a constant, provided that the integral exists. By $\|x_t\|_2$ we mean that $\|x_t\|_{2\delta}$ with $\delta = 0$, and we say that $x \in \mathcal{L}_{2e}$ if $\|x_t\|_2$ exists. Let $x \in \mathcal{L}_{2e}$, and consider the set

$$\mathcal{S}(\mu) = \left\{ x : \int_t^{t+T} |x(\tau)|^2 d\tau \leq c_0 \mu T + c_1,\ \forall t,\ T \geq 0 \right\}$$

for a given constant μ, where $c_0, c_1 \geq 0$ are some finite constants, and c_0 is independent of μ. We say that x is μ-small in

[1]State resetting schemes may reduce transients after switching. Adaptive mixing control is presented as an alternative approach that aims to avoid state-resetting due to unnecessary switching.

the mean square sense (m.s.s.) if $x \in \mathcal{S}(\mu)$. Furthermore, consider the signal $w : [0, \infty) \rightarrow \mathbb{R}^+$ and the set

$$\mathcal{S}(w) = \left\{ y : \int_t^{t+T} y^T(\tau) y(\tau) d\tau \le c_0 \int_t^{t+T} w(\tau) d\tau + c_1, \quad \forall t, T \ge 0 \right\}$$

where $c_0, c_1 \ge 0$ are some finite constants. We say that x is w-small in the m.s.s. if $x \in \mathcal{S}(w)$.

Let $H(s)$ and $h(t)$ be the transfer function and impulse response, respectively, of some LTI system. If $H(s)$ is a proper transfer function and analytic in $\mathcal{R}e[s] \ge -\delta/2$ for some $\delta \ge 0$, where $\mathcal{R}e[s]$ denotes the real part of s, then the $\mathcal{H}_\infty$ system norm is given by $\|H\|_\infty \triangleq \sup_{j\omega} |H(j\omega)|$. The $\|\cdot\|_{2\delta}$ system norm of $H(s)$ is defined as $\|H\|_{2\delta} \triangleq 1/\sqrt{2\pi} \left\{ \int_{-\infty}^{\infty} |H(j\omega - \delta/2)|^2 d\omega \right\}^{1/2}$. The induced $\mathcal{L}_\infty$ system norm of H is given by $\|H\|_{\infty - gn} = \|h\|_1$. If $y = H(s)u$ and $u \in \mathcal{L}_\infty$ then $\|y\|_\infty \le \|H\|_{\infty - gn} \|u\|_\infty$.

If $X, Y \subset \mathbb{R}^m$, δX denotes the boundary of the set X, and $X - Y$ denotes the set-theoretic difference. An open ball of radius r centered around point $x_0 \in X$ is denoted as $B_r(x_0) = \{x \in \mathbb{R}^m : |x - x_0| < r\}$. Throughout, $e_i \in \mathbb{R}^p$ denotes the i^{th} standard basis vector, i.e., the i^{th} component of e_i is one; all other components are zero. The function $\psi : \mathbb{R} \rightarrow [0, 1]$ denotes the *bump function* $\psi(x) = e^{-1/1-x^2}$ if $|x| < 1$; otherwise, $\psi(x) = 0$. The function $\psi(x)$ is smooth and supported on $(-1, 1)$.

We say that a square, bounded, piece-wise continuous $A(t)$ is exponentially stable (e.s.) if its transition matrix $\Phi(t, \tau)$ satisfies $\|\Phi(t, \tau)\| \le \lambda_0 e^{-\alpha_0 (t - \tau)}$ for some $\lambda_0, \alpha_0 > 0$ for all $t \ge \tau \ge 0$.

Theorem 1: Let $\Omega \subset \mathbb{R}^{2n}$ be compact and θ_0 be any constant in Ω and $\theta : \mathbb{R}^+ \rightarrow \mathbb{R}^{2n}$. If the parameterized detectable pair $(C(\theta_0), A(\theta_0))$ is continuously differentiable with respect to $\theta_0 \in \Omega$, where $A(\theta_0) \in \mathbb{R}^{n \times n}$ and $C(\theta_0) \in \mathbb{R}^{l \times n}$, then there exists a continuously differentiable matrix function $L : \Omega \rightarrow \mathbb{R}^{n \times l}$, such that

1) if $\theta(t) \in \Omega$ for all $t \ge 0$ and $\dot{\theta} \in \mathcal{L}_2$, then the equilibrium $x_e = 0$ of $\dot{x} = A_I(t) x$ is e.s.
2) if $\theta(t) \in \Omega$ for all $t \ge 0$ and $\dot{\theta} \in \mathcal{S}(\mu^2)$ for some constant $\mu > 0$, then there exists a $\mu^* > 0$ such that if $\mu \in [0, \mu^*)$ the equilibrium $x_e = 0$ of $\dot{x} = A_I(t) x$ is e.s.

where $A_I(t) \triangleq A(\theta(t)) - L(\theta(t)) C(\theta(t))$. □

The proof of Theorem 1 is a combination of the well-known results of [26] and the linear time varying (LTV) stability results found in [5, Theorem 3.4.11]. We would like to emphasize that the existence of the observer gain L of Theorem 1 is utilized only for analysis purposes, not design. Furthermore, an explicit definition of L is not required.

II. A Simple Example

In this section, we use a simple example to introduce the adaptive mixing control approach in a tutorial manner. Consider the uncertain plant

$$y = \frac{1}{s - \theta^*} (1 + \Delta_m(s))(u + d), \quad \theta^* \in \Omega = [-2.5, 2.5] \quad (1)$$

where θ^* is an unknown constant that belongs to the known interval Ω; d is a bounded disturbance, i.e., $|d(t)| \le d_0 \; \forall t \ge 0$; and $\Delta_m(s)$ is a multiplicative plant uncertainty. $\Delta_m(s)$ is assumed to be a proper rational transfer function that is analytic in $\mathcal{R}e[s] \ge -\delta_0/2$ for some known $\delta_0 > 0$. We refer to $G_0(s) = 1/(s - \theta^*)$ as the nominal model. The control objective is to place the pole of the closed-loop nominal plant in the interval $[-5, -3]$; guarantee that y and u are bounded; and when $d_0 = 0$, guarantee y and u converge to zero as $t \rightarrow \infty$.

We consider control laws of the form $u = -ky$, where k is to be chosen such that the control objective is met for any $\theta^* \in [-2.5, 2.5]$. The question is whether a single fixed value of k will meet the control objective given that θ^* is unknown except that it belongs to the interval $[-2.5, 2.5]$. If we apply the above control law to the plant (1), we obtain the closed-loop system

$$y = \frac{1}{s - \theta^* + (1 + \Delta_m(s))k} (1 + \Delta_m(s)) d. \quad (2)$$

When $\Delta_m = 0$, the closed-loop pole of (2) is $s = \theta^* - k$, and it follows from the bound $-2.5 \le \theta^* \le 2.5$ that a single fixed value of k cannot meet the control objective.

Let us assume that the large parametric uncertainty $\Omega = [-2.5, 2.5]$ is divided into the three smaller subintervals

$$\Omega_1 = [0.5, 2.5], \quad \Omega_2 = [-1, 1], \quad \Omega_3 = [-2.5, -0.5]$$

called *parameter subsets*, so that the control objective could be met if it is known which parameter subset contains θ^*. Let us also assume that the following three fixed controllers (*candidate controllers*) are used based on which parameter subset contains θ^*:

$$u = -k_1 y, \quad k_1 = 5.5, \quad \text{if } \theta^* \in \Omega_1 \quad (3)$$
$$u = -k_2 y, \quad k_2 = 4, \quad \text{if } \theta^* \in \Omega_2 \quad (4)$$
$$u = -k_3 y, \quad k_3 = 2.5, \quad \text{if } \theta^* \in \Omega_3. \quad (5)$$

If θ^* is known *a priori* then the control objective of placing the nominal closed-loop pole in the interval $[-5, -3]$ is met by selecting the appropriate candidate control law from (3)–(5) based on which parameter subset contains θ^*. If θ^* belongs to more than one parameter subset, then any of the appropriate candidate control laws may be chosen. Furthermore, overall closed-loop stability is guaranteed if

$$\|\Delta_m\|_\infty < \frac{1}{k_i} \left\| \frac{1}{s - \theta^* + k_i} \right\|_\infty^{-1}, \quad \forall \theta^* \in \Omega_i, \quad i = 1, 2, 3.$$

The right hand side of the above takes on its minimum value over Ω when $\theta^* = 2.5 \in \Omega_1$ and $k = k_1 = 5.5$. Thus, a sufficient condition for stability is $\|\Delta_m\|_\infty < 3/5.5$.

If θ^* is not known *a priori* but is measured or estimated online, a strategy is needed for constructing a control law that meets the control objective based on the real-time knowledge of θ^*. Our objective is to design a control law $u = -k(\theta^*) y$ from the candidate control laws (3)–(5) continuous in θ^* to avoid discontinuities in the control signal that may occur as a result of noise or if the measured value of θ^* varies across parameter subsets. Discontinuities in the control signal are a practical concern

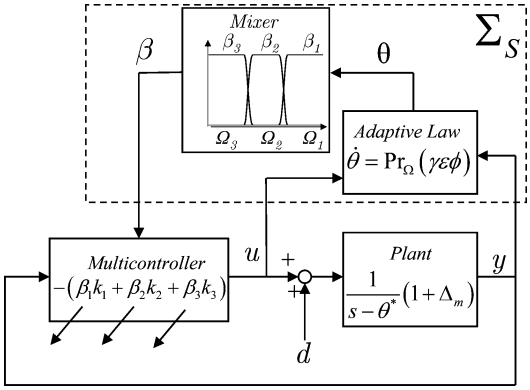

Fig. 2. Block diagram of adaptive mixing control scheme.

because it may lead to control chattering or poor transient performance.

Because the simple control law $u = -(\theta^* + 3)y$ achieves the control objective, it may not be obvious why we seek to construct the overall controller $k(\theta^*)$ from the candidate controllers $k_1,\ k_2,\ k_3$ and their corresponding parameter subsets $\Omega_1,\ \Omega_2,\ \Omega_3$. In general, the unknown parameter θ^* may not enter into the control law gains in a tractable manner (e.g., as in the case of μ-synthesis), making it difficult in the adaptive case to compute online the controller gains given the estimate $\theta(t)$. However, given the candidate controllers and parameter subsets, it is straightforward to assign θ^* (or $\theta(t)$ in the adaptive case) a stabilizing candidate controller by testing which parameter subsets contain θ^*.

We follow the latter principle to construct $k(\theta^*)$ and develop an approach called *control mixing* where the candidate control laws are weighted based on the real-time information on θ^*. The *parameter overlaps*

$$\Omega_1 \cap \Omega_2 = [0.5,\ 1], \quad \Omega_2 \cap \Omega_3 = [-1, -0.5] \tag{6}$$

provide a domain in which the controller weights can be varied to continuously transition from one controller to another. The method for this simple example is described below. In Section V we extend the method to a general plant, and in Section VI a more complicated example involving mixed-μ compensators is considered.

Let us consider a control law of the form

$$\begin{aligned} u &= -k(\theta^*)y, \\ k(\theta^*) &= \beta_1(\theta^*)k_1 + \beta_2(\theta^*)k_2 + \beta_3(\theta^*)k_3 \end{aligned} \tag{7}$$

where $k_1 = 5.5$, $k_2 = 4$, and $k_3 = 2.5$ are stabilizing gains if $\theta^* \in \Omega_1, \Omega_2, \Omega_3$, respectively, and $\beta_1,\ \beta_2,\ \beta_3$ are weights to be chosen so that for any $\theta^* \in \Omega$ the conditions

$$\beta_i(\theta^*) = 0 \quad \text{if } \theta^* \notin \Omega_i, \quad i = 1,2,3 \tag{8}$$

$$\beta_1(\theta^*), \beta_2(\theta^*), \beta_3(\theta^*) \geq 0 \tag{9}$$

$$\beta_1(\theta^*) + \beta_2(\theta^*) + \beta_3(\theta^*) = 1 \tag{10}$$

are satisfied.

We now verify that the control law (7), with (8)–(10) satisfied, guarantees that the control objective is met. If θ^* belongs to only one parameter subset Ω_i then it follows from (8)–(10) that the control law is $u = -k_i y$, which has been constructed to meet the control objective. What remains is to establish that the control objective is met on the parameter overlaps (6). If θ^* belongs to the model overlap $\Omega_1 \cap \Omega_2 = [0.5,\ 1]$ then the control law is of the form $u = -(\beta_1 k_1 + \beta_2 k_2)y$, where $k_1 = 5.5$, $k_2 = 4$, and $\beta_1, \beta_2 \geq 0$ are some constants that satisfy $\beta_1 + \beta_2 = 1$. Thus, the closed-loop pole $s = \theta^* - k$ satisfies $-5 \leq s \leq -3$, and the control objective is met. A similar analysis shows that the control objective is also satisfied for $\theta^* \in \Omega_2 \cap \Omega_3$. Therefore, if the conditions (8)–(10) are satisfied, the mixing control law (7) satisfies the control objective for every θ^* belonging to $\Omega = [-2.5,\ 2.5]$.

The next step is to define $\beta_i (i = 1,2,3)$ such that (8)–(10) are satisfied. For any $\theta^* \in \Omega$, consider

$$\beta_i(\theta^*) = \frac{\tilde{\beta}_i(\theta^*)}{\tilde{\beta}_1(\theta^*) + \tilde{\beta}_2(\theta^*) + \tilde{\beta}_3(\theta^*)}, \quad i = 1,2,3 \tag{11}$$

where $\tilde{\beta}_1(\theta^*) = \psi(\theta^* - 1.75/1.25)$, $\tilde{\beta}_2(\theta^*) = \psi(\theta^*)$, $\tilde{\beta}_3(\theta^*) = \psi(\theta^* + 1.75/1.25)$, and ψ is the smooth bump function (cf. Section II). We define the *mixing signal* as $\beta \triangleq [\beta_1\ \beta_2\ \beta_3]^T$. Now we establish whether the mixing signal (11) satisfies requirements (8)–(10). Because ψ is supported on $(-1,\ 1)$, $\tilde{\beta}_1$ is supported on $(0.5,\ 3)$; $\tilde{\beta}_2$ is supported on $(-1,\ 1)$; and $\tilde{\beta}_3$ is supported on $(-3, -0.5)$. Thus, (8)–(10) are satisfied for any $\theta^* \in \Omega$. Because the mixing signal (11) satisfies conditions (8)–(10), the parameterized control law given by (7) and (11) meets the control objective.

We now analyze the robustness of the mixing control scheme (7), (11) applied to the true plant $G(s) = (1 + \Delta_m(s))G_0(s)$. It follows from the Nyquist stability criterion that a sufficient condition for stability is:

$$\|\Delta_m\|_\infty < \frac{1}{k(\theta^*)} \left\| \frac{1}{s - \theta^* + k(\theta^*)} \right\|_\infty^{-1}.$$

The right hand side takes on its minimum value over Ω when $\theta^* = 2.5$, yielding the sufficient condition $\|\Delta_m\|_\infty < 3/5.5$ for the closed-loop stability.

The implementation of the mixing control law (7) requires that θ^* is a known constant. In application, this knowledge may come by monitoring certain auxiliary signals, or it may be based on the results of an online parameter estimator, which is the approach of this paper. The parameter estimator is designed by following the procedures of [5, Sec. 2.4.1]. We rewrite the nominal plant model $\dot{y} = \theta^* y + u$ as $\bar{z} = \theta^* \bar{\phi}$, where $\bar{z} = \dot{y} - u$ and $\bar{\phi} = y$. If $\bar{z}$ and $\bar{\phi}$ were available for measurement, we could generate the estimation error $\epsilon_1 = \bar{z} - \hat{\bar{z}} = \bar{z} - \theta\bar{\phi}$, given the estimate θ of θ^*. Because $\dot{y}$ cannot be reliably measured, this definition of ϵ_1 is not implementable. Therefore, we filter both sides of $\bar{z} = \theta^* \bar{\phi}$ by the stable filter $F(s) = \lambda/(s + \lambda)$, where $\lambda > 0$, to develop the linear parametric model (LPM)

$$z = \theta^* \phi, \quad z = \frac{\lambda s}{s + \lambda} y - \frac{\lambda}{s + \lambda} u, \quad \phi = \frac{\lambda}{s + \lambda} y$$

where z and ϕ are generated by filtering y and u. Given the estimate θ, the estimation error is generated by $\epsilon_1 = z - \theta\phi$, which will be used to drive the adaptive law, whose task is to

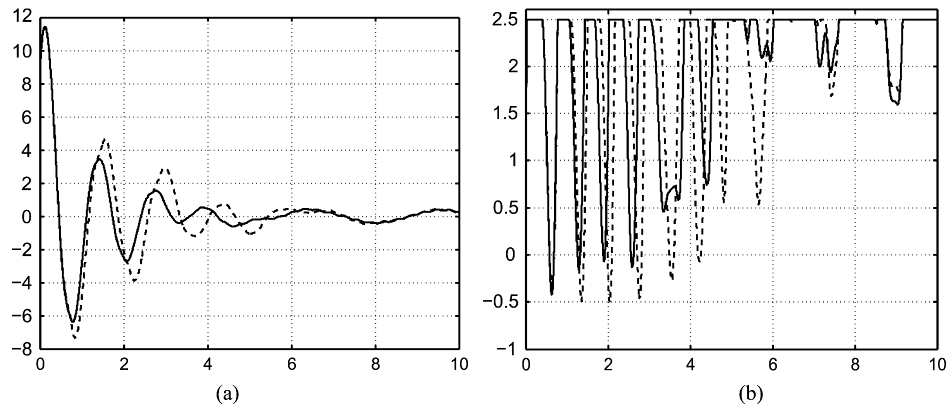

Fig. 3. Simulation results. (a) Plant output: Adaptive mixing control (solid); Adaptive pole placement control (dashed). (b) Estimate $\theta(t)$ of $\theta^* = 2.5$: Adaptive mixing control (solid); Adaptive pole placement control (dashed).

tune θ to make ϵ_1 "small." This LPM can be used to generate a wide-class of adaptive laws for generating the estimate θ of θ^* [5, Section 8.5], and we choose the adaptive law as the gradient algorithm with projection modification

$$\begin{aligned}\dot{\theta} &= \mathrm{Pr}_\Omega(\gamma\epsilon\phi) \\ &= \begin{cases}\gamma\epsilon\phi, & |\theta| < 2.5 \text{ or } \gamma\epsilon\phi \operatorname{sgn}\theta \le 0 \\ 0, & \text{otherwise}\end{cases} \qquad (12)\\ \epsilon &= \frac{\epsilon_1}{m^2} = \frac{z - \theta\phi}{m^2}, \quad m^2 = 1 + n_d, \\ \dot{n}_d &= -\delta_0 n_d + u^2 + y^2\end{aligned}$$

where $\gamma > 0$ is the adaptive gain and Pr_Ω is the projection operator that restricts $\theta(t)$ to Ω. We refer to $\epsilon_1 = z - \theta\phi$ as the *unnormalized* estimation error to distinguish it from the *normalized* estimation error $\epsilon = \epsilon_1/m^2$. Completing the adaptive mixing control design, we combine the adaptive law (12) with $k(\theta^*)$ by replacing θ^* with its estimate $\theta(t)$, and the adaptive mixing control scheme is shown in Fig. 2.

A. Simulation

For simulation purposes, we use the plant parameters $\theta^* = 2.5 \in \Omega_1$, $\Delta_m(s) = -2\mu s/(1+\mu s)$, $\mu = 0.1$, and $d(t) = \cos 2t$. Additionally, sensor noise $\nu(t) = 0.1\cos(20t)$ is simulated by substituting the noisy measurement $y_m(t) = y(t)+\nu(t)$ for y in the control law and parameter estimator. We use the control parameters $\gamma = 100$, $\lambda = 5$, $\delta_0 = 4$, and $\theta(0) = 0$, which were chosen by trial and error. For comparison, we also simulate an adaptive pole placement control (APPC) scheme that uses an identical parameter estimator as the adaptive mixing control scheme. The APPC scheme differs from the adaptive mixing control scheme only in the control law, where the APPC scheme replaces the mixing control law (7) with the APPC control $u = -(\theta(t)+3)y$. Moreover, the desired pole location $s = -3$ of the APPC control law was chosen from robustness considerations. The plant output is shown in Fig. 3(a). In this example, both the adaptive mixing control and APPC schemes regulate the plant output towards a neighborhood of zero, where the adaptive mixing scheme has slightly faster convergence. Simulations show that the APPC scheme remains stable for $\mu \le 0.113$; adaptive mixing control remains stable for $\mu \le 0.12$; and perfect identification $\theta(t) \equiv \theta^*$ remains stable for $\mu < 0.125$. Adaptive mixing control's mild improvement in performance and robustness is because oscillations in θ (shown in Fig. 3(b)) caused by the modeling error terms Δ_m, $d(t)$, and $\nu(t)$ do not affect the control law when $\theta(t) \ge 0.5$. This is not the case in APPC; thus, oscillations in u caused by θ further excite Δ_m. Because both schemes use an adaptive law with projection modification, the online estimates remain bounded by $-2.5 \le \theta(t) \le 2.5$, which together with the large model uncertainty gives rise to the "sawtooth" shaped $\theta(t)$ trajectories shown in Fig. 3(b).

III. General Problem Formulation

Consider the SISO LTI plant

$$\begin{aligned}y &= G(s;\theta^*)u + d, \\ G(s;\theta^*) &= G_0(s;\theta^*)(1+\Delta_m(s)) \qquad (13)\\ G_0(s) &= \frac{N_0(s)}{D_0(s)} = \frac{\theta_b^{*T}\alpha_{n-1}(s)}{s^n + \theta_a^{*T}\alpha_{n-1}(s)} \qquad (14)\\ y_m &= y + \nu \qquad (15)\end{aligned}$$

where $G_0(s)$ represents the nominal plant; the vector $\theta^* \triangleq [\theta_b^{*T}\ \theta_a^{*T}]^T \in \Omega \subset \mathbb{R}^{2n}$ contains the unknown parameters of $G_0(s;\theta^*)$; $\alpha_{n-1}(s) \triangleq [s^{n-1}\ s^{n-2}\ \cdots\ 1]^T$; y_m is the measured value of y corrupted by the bounded sensor noise ν, i.e., $|\nu(t)| \le \nu_0,\ \forall t \ge 0$; $\Delta_m(s)$ is an unknown multiplicative perturbation; and d is a bounded disturbance, i.e., $|d(t)| \le d_0,\ \forall t \ge 0$; The control objective is to choose the plant input u so that the plant output y is regulated close to zero. We make the following assumptions on the plant to meet the control objective:

P1. $D_0(s)$ is a monic polynomial whose degree n is known.

P2. Degree $(N_0) < n$.

P3. $\Delta_m(s)$ is proper, rational, and analytic in $\mathcal{R}e[s] \ge -\delta_0/2$ for some known $\delta_0 > 0$.

P4. $\theta^* \in \Omega$ for some known compact convex set $\Omega \subset \mathbb{R}^{2n}$.

Consider the state-space realization of (13)

$$\dot{x}_{\mathbb{P}} = A_{\mathbb{P}}x_{\mathbb{P}} + B_{\mathbb{P}}u, \quad y_m = C_{\mathbb{P}}x_{\mathbb{P}} + d + \nu. \qquad (16)$$

We make the additional assumption to make control meaningful:

P5. The pairs $(C_{\mathbb{P}},\ A_{\mathbb{P}})$ and $(A_{\mathbb{P}},\ B_{\mathbb{P}})$ are detectable and stabilizable, respectively, on Ω.

It should be emphasized that both unstable and nonminimum phase plants are admissible despite requirements P1-P5.

Given is a family of p candidate controllers $\mathcal{C} \triangleq \{K_i(s)\}_{i\in\mathcal{I}}$, where $K_1(s),\ldots,K_p(s)$ are rational transfer functions and $\mathcal{I}$ denotes the index set $\{1,\ldots,p\}$, and the parameter partition $\mathcal{P} \triangleq \left\{\Omega_i \subset \mathbb{R}^{2n}\right\}_{i\in\mathcal{I}}$, where each parameter subset Ω_i is compact and $\mathcal{P}$ covers Ω, i.e., $\Omega \subset \cup_{i\in\mathcal{I}}\Omega_i$. The candidate controller set $\mathcal{C}$ and parameter partition $\mathcal{P}$ has been developed such that for every $i \in \mathcal{I}$ and each $\theta^* \in \Omega_i$, the control law $u = -K_i(s)y_m$ yields a stable closed-loop system that meets some performance requirements. If Ω_i contains θ^* and the control is chosen as $u = -K_i(s)y_m$, then a sufficient condition for stability over Ω is

$$\|\Delta_m\|_\infty \leq \min_{\theta^*\in\Omega_i} \left\| \frac{K_i(s)G(s;\theta^*)}{1+K_i(s)G(s;\theta^*)} \right\|_\infty^{-1}. \tag{17}$$

Let $\delta\Omega_i$ and $\delta\Omega$ be the boundaries of the sets Ω_i and Ω, respectively. To facilitate control mixing, the partition has an *overlapping property*: for all $i \in \mathcal{I}$ and any $\theta^* \in (\delta\Omega_i - \delta\Omega)$ there exist constants $r > 0$ and $j \neq i$ such that $B_r(\theta^*) \subset \Omega_j$. The parameter overlaps set $\mathcal{O}$ is the set of all points $\theta^* \in \Omega$ that belong to more than one parameter subset. The candidate controllers and parameter subsets can, for example, be generated by the method of [19], modified to generate overlapping parameter subsets.

Remark 1: A conventional adaptive controller could be included in the candidate controller set $\mathcal{C}$ to account for the case $\theta^* \notin \Omega$. This is the topic of future work.

Remark 2: As in many MMAC approaches, the complexity of the controller design increases with the number of candidate controllers, which may result in an impractical design. *A priori* knowledge can be used to decrease the number of unknown parameters and size of Ω, and, in turn, decrease the number of candidate controllers.

1) The Problem: The objective of this paper is to propose a provably correct MMAC scheme which is capable of achieving 1) global boundedness of all system signals and 2) regulation of all plant signals in the absence of unmodeled dynamics, disturbances, and sensor noise.

A deterministic approach is pursued because the disturbance is only known to be bounded. The unknown parameter vectors θ_a^*, θ_b^* enter the plant model linearly and the plant remains detectable and stabilizable on Ω. Therefore, in order to side-step issues associated with discontinuous switching among candidate controllers, we present a deterministic MMAC approach that tunes the multicontroller in a continuous manner based on the estimate $\theta(t)$ of θ^*.

IV. Conceptual Framework

The adaptive mixing control architecture comprises two systems: the multicontroller $\mathbb{C}(\beta,\theta)$ and the robust adaptive supervisor Σ_S, which in turn consists of a robust parameter estimator and mixer M. For notational simplicity, the following definitions are made. If $\theta \in \Omega_i$ (for some $i \in \mathcal{I}$), then Ω_i is said to be an *active parameter subset*. $\mathcal{I}_\theta$ denotes the *index set of all active parameter subsets at* $\theta \in \Omega$, i.e., $\mathcal{I}_\theta = \{i \in \mathcal{I} : \theta \in \Omega_i\}$. We define the *set of all admissible mixing values at* $\theta \in \Omega$ as $\mathcal{B}_\theta \triangleq \{\beta = [\beta_1 \cdots \beta_p]^T \in [0,\, 1]^p\colon \sum_{i\in\mathcal{I}}\beta_i = 1;\ \beta_i = 0,\ i \notin \mathcal{I}_\theta\}$. The *set of all admissible mixing values* $\cup_{\theta\in\Omega}\mathcal{B}_\theta$ is denoted by $\mathcal{B}$.

A. Multicontroller

The multicontroller $\mathbb{C}(\beta,\theta)$, constructed from $\mathcal{C}$, is a dynamical system capable of generating a mix of candidate control laws. The multicontroller $\mathbb{C}(\beta,\theta)$ is given by the stabilizable and detectable state-space realization

$$\dot{x}_\mathbb{C} = A_\mathbb{C}(\beta,\theta)x_\mathbb{C} + B_\mathbb{C}(\beta,\theta)y_m, \quad u = -C_\mathbb{C}(\beta,\theta)x_\mathbb{C} \tag{18}$$

where $x_\mathbb{C}(t) \in \mathbb{R}^{n_c}$ is the multicontroller state vector; the system matrices $A_\mathbb{C}$, $B_\mathbb{C}$, $C_\mathbb{C}$ are of compatible dimensions; and the mixing signal $\beta(t) \triangleq [\beta_1(t) \cdots \beta_p]^T$ and estimate $\theta(t)$ is generated by the supervisor Σ_S and tunes $\mathbb{C}$. For fixed values of $\beta \in \mathcal{B}$ and $\theta \in \Omega$, the multicontroller $u = -K(s;\beta,\theta)y_m$ has the transfer function

$$K(s;\beta,\theta) = C_\mathbb{C}(\beta,\theta)(sI - A_\mathbb{C}(\beta,\theta))^{-1}B_\mathbb{C}(\beta,\theta) \tag{19}$$

$$= \frac{N_K(s;\beta,\theta)}{D_K(s;\beta,\theta)}. \tag{20}$$

The multicontroller $\mathbb{C}(\beta,\theta)$ satisfies three properties:

C1. The elements of $A_\mathbb{C}(\beta,\theta)$, $B_\mathbb{C}(\beta,\theta)$, and $C_\mathbb{C}(\beta,\theta)$ are continuously differentiable in β and θ.

C2. $K(s;e_i,\theta) = K_i(s)$, where $\theta \in \Omega_i$ and $e_i \in \mathbb{R}^p$ is i^{th} standard basis vector.

C3. For all $\theta^* \in \Omega$ and any $\beta^* \in \mathcal{B}_{\theta^*}$, $K(s;\beta^*,\theta^*)$ internally stabilizes the plant.

Property C1 ensures that the closed-loop system varies slowly if β and θ are tuned slowly. Property C2 allows for each candidate controller to be recovered. Property C3 ensures that $\mathbb{C}(\beta,\theta)$ is a stabilizing certainty equivalence control law for any admissible mixing signal, independent of the mixer implementation. The multicontroller can be viewed as a generalization of the multicontroller used in supervisory control [6].

Construction of the multicontroller involves interpolating the candidate controllers over the parameter overlaps $\mathcal{O}$. Numerous controller interpolation approaches have been proposed in the context of gain scheduling. These methods interpolate controller poles, zeros, and gains [27]; solutions of the Riccati equations for an $\mathcal{H}_\infty$ design [28]; state-space coefficient matrices of balanced controller realizations [29]; state and observer gains [30]; controller output [31]–[33], i.e., $u = \sum_{i\in\mathcal{I}}\beta_i u_i$, where $u_i = -K_i(s)y_m$. As in gain scheduling, these interpolation methods may not satisfy the point-wise stability requirement C3 (cf. the counter examples of [34], [35]). Thus, if one of these interpolation methods is used then property C3 should be verified.

Fortunately, there also exist theoretically justified methods [34]–[36], which can be used to construct the multicontroller. The following result is adapted from [34]:

Lemma 2: Consider the nominal plant $G_0(s;\theta^*)$ given by (14) and the stable coprime factorization $G_0(s;\theta^*) = N(s;\theta^*)M^{-1}(s;\theta^*) = \tilde{M}^{-1}(s;\theta^*)\tilde{N}(s;\theta^*)$ that depends on θ^* smoothly. Let the stable transfer functions $X(s;\theta^*)$, $Y(s;\theta^*)$, $\tilde{X}(s;\theta^*)$, and $\tilde{Y}(s;\theta^*)$ depend on θ^* smoothly and satisfy the double Bezout identity (omitting dependencies on s and θ^*)

$$\begin{aligned}\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} &= \begin{bmatrix} \tilde{X} & \tilde{Y} \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & -Y \\ N & X \end{bmatrix} \\ &= \begin{bmatrix} M & -Y \\ N & X \end{bmatrix} \begin{bmatrix} \tilde{X} & \tilde{Y} \\ -\tilde{N} & \tilde{M} \end{bmatrix}, \quad \forall \theta^* \in \Omega.\end{aligned}$$

Suppose $\theta^* \in \bar{\Omega}$, where $\bar{\Omega} \subset \Omega$ is some compact set, and $K_1(s), \ldots, K_{\bar{p}}(s)$ are $\bar{p}$ stabilizing negative feedback controllers for any $\theta^* \in \bar{\Omega}$. Let each controller $K_i(s)$ be given a stable coprime factorization $K_i(s) = \bar{U}_i(s)\bar{V}_i^{-1}(s)$, and we define

$$\begin{aligned} Q_i(s;\theta^*) = &\left(\tilde{Y}(s;\theta^*)\bar{V}_i(s) - \tilde{X}(s;\theta^*)\bar{U}_i(s)\right) \\ &\cdot \left(\tilde{N}(s;\theta^*)\bar{U}_i(s) + \tilde{M}(s;\theta^*)\bar{V}_i(s)\right)^{-1} \end{aligned} \tag{21}$$

$$U_i(s;\theta^*) = M(s;\theta^*)Q_i(s;\theta^*) - Y(s;\theta^*) \tag{22}$$

$$V_i(s;\theta^*) = N(s;\theta^*)Q_i(s;\theta^*) - X(s;\theta^*) \tag{23}$$

for $i = 1, \ldots, \bar{p}$. If the controller is given by

$$K(s;\bar{\beta},\theta^*) = U(s;\bar{\beta},\theta^*)V^{-1}(s;\bar{\beta},\theta^*), \quad \bar{\beta} = \left[\bar{\beta}_1 \cdots \bar{\beta}_{\bar{p}}\right]^T \tag{24}$$

where $U(s) = \sum_{i=1}^{\bar{p}} \bar{\beta}_i U_i(s;\theta^*)$, $V(s) = \sum_{i=1}^{\bar{p}} \bar{\beta}_i V_i(s;\theta^*)$, and $\bar{\beta}_1, \ldots, \bar{\beta}_{\bar{p}}$ are some positive constants that satisfy $\sum_{i=1}^{\bar{p}} \bar{\beta}_i = 1$, then $K(s)$ is stabilizing and $K(s) = K_i(s)$ if $\bar{\beta}_j = 1$ for $j = i$ and $\bar{\beta}_j = 0$ for $j \neq i$. □

Motivated by Lemma 2, consider the multicontroller

$$K(s;\beta,\theta) = U(s;\beta,\theta)V^{-1}(s;\beta,\theta) \tag{25}$$

$$U(s;\beta,\theta) = \sum_{i=1}^{p} \beta_i U_i(s;\varphi_{\Omega_i}(\theta)) \tag{26}$$

$$V(s;\beta,\theta) = \sum_{i=1}^{p} \beta_i V_i(s;\varphi_{\Omega_i}(\theta)) \tag{27}$$

where Q_i, U_i, and V_i are given by (21)–(23), respectively, and the function $\varphi_{\Omega_i} : \Omega \to \Omega_i$ denotes a smooth projection function[2]: φ_{Ω_i} is continuously differentiable and $\varphi_{\Omega_i}(\theta) = \theta$ if $\theta \in \Omega_i$. The function φ_{Ω_i} is used to ensure that each Q_i is only evaluated on Ω_i and, therefore, stable; otherwise, Q_i may generate an unbounded out-of-the-loop signal if $\beta_i = 0$ and $\theta \notin \Omega_i$.

We now examine if this multicontroller satisfies properties C1-C3. Property C1 follows immediately by inspection of the filters of Lemma 2 that define the multicontroller $K(s;\beta,\theta^*)$ and because φ_{Ω_i} is smooth. Let us now consider property C3. For fixed constants $\theta^* \in \Omega$ and $\beta^* \in \mathcal{B}_{\theta^*}$, we have $\beta_i^* = 0$ for $i \notin \mathcal{I}_{\theta^*}$, and $\varphi_{\Omega_i}(\theta^*) = \theta^*$ for $i \in \mathcal{I}_{\theta^*}$. Thus

$$\begin{aligned} K(s;\beta^*,\theta^*) = &\left(\sum_{i\in\mathcal{I}_{\theta^*}} \beta_i^* U_i(s;\theta^*)\right) \\ &\times \left(\sum_{i\in\mathcal{I}_{\theta^*}} \beta_i^* V_i(s,\theta^*)\right)^{-1}. \end{aligned} \tag{28}$$

[2]φ_{Ω_i} is not the same as the projection operator $\mathrm{Pr}_{\Omega_i}\{\cdot\}$ used in the adaptive law. φ_{Ω_i} can be constructed, for example, from smooth bump functions ψ.

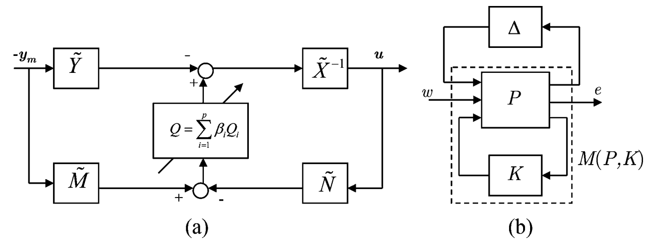

Fig. 4. Multicontroller implementation and robust performance formulation (a) Multicontroller structure (b) General control configuration.

Because $\mathcal{I}_{\theta^*}$ indexes only stabilizing controllers, it follows from Lemma 2 that $K(s;\beta^*,\theta^*)$ internally stabilizes the plant, satisfying C3. Moreover, if $\beta^* = e_i$ for some $i \in \mathcal{I}_{\theta^*}$, $K(s,\beta^*,\theta^*) = K_i(s)$, and C2 is satisfied. Fig. 4(a) shows an implementation of the multicontroller that avoids unbounded out-of-the-loop signals [37].

By *Q-blending* we mean the multicontroller scheme (25), which is given in a multiple input-multiple output (MIMO) format to emphasize that this approach is suitable for extensions of adaptive mixing control to the MIMO case.

We now consider robust performance as the control objective, which is formulated in the standard general control configuration shown in Fig. 4(b), as the control objective. Motivated to work with a normalized uncertainty, we model the multiplicative uncertainty as $\Delta_m(s) = w_m(s)\Delta(s)$, where the known stable transfer function $w_m(s)$ serves as a frequency dependent weight and $\Delta(s)$ is any stable transfer function satisfying $\|\Delta\|_\infty \leq 1$. The transfer matrix $P(s;\theta^*)$ is assumed to contain the plant $G_0(s;\theta^*)$ and its interconnection with the performance and uncertainty weights that describe the control objective; $w(t) \in \mathbb{R}^{n_w}$ is the exogenous input (for example, say $w = [d\ \nu]^T$), and $e(t) \in \mathbb{R}^{n_e}$ is the exogenous output. The matrix transfer function $M(P(\theta^*), K)$ is formed by absorbing the controller K into P as shown in Fig. 4(b). We say that a controller K yields *robust performance* if $M(P(\theta^*), K)$ is stable and satisfies $\|M(P(\theta^*), K)\|_{\hat{\Delta}} < 1$, where $\hat{\Delta} \triangleq \mathrm{diag}(\Delta, \Delta_P)$ satisfies $\|\hat{\Delta}\|_\infty \leq 1$; Δ_P is a stable $n_w \times n_e$ transfer matrix; and $\|M\|_{\hat{\Delta}}$ denotes $\sup_{\omega\in\mathbb{R}} \mu_{\hat{\Delta}}(M(j\omega))$. Let us assume that each candidate $K_i(s)$ yields robust performance for all $\theta^* \in \Omega_i$. As demonstrated by the following result, a Q-blending multicontroller preserves robust performance.

Lemma 3: If the multicontroller $K(\beta,\theta^*)$ is given by (25) and, for some generalized plant P and $i = 1, \ldots, p$, the candidate controller K_i yields robust performance for all $\theta^* \in \Omega_i$, then for all $\theta^* \in \Omega$ and $\beta^* \in \mathcal{B}_{\theta^*}$ the multicontroller $K(\beta^*,\theta^*)$ yields robust performance with respect to P. □

Proof: The transfer matrix M has the form $M = M_1 + M_2QM_3$ (cf. [2, pp. 150]), where $Q = \sum_{i=1}^{p} \beta_i^* Q_i$. For any $\theta^* \in \Omega$ and $\beta^* \in \mathcal{B}_{\theta^*}$, it follows that $\sum_{i\in\mathcal{I}_{\theta^*}} \beta_i^* = 1$, $\beta_i^* = 0$ for $i \notin \mathcal{I}_{\theta^*}$, and $\|M(P,K_i)\|_{\hat{\Delta}} < 1$ for $i \in \mathcal{I}_{\theta^*}$. Thus

$$\begin{aligned} M(P,K(\beta^*,\theta^*)) &= M_1 + M_2 \sum_{i\in\mathcal{I}_{\theta^*}} \beta_i^* Q_i M_3 \\ &= \sum_{i\in\mathcal{I}_{\theta^*}} \beta_i^* (M_1 + M_2 Q_i M_3) \\ &= \sum_{i\in\mathcal{I}_{\theta^*}} \beta_i^* M(P,K_i) \end{aligned}$$

and, consequently, $M(P, K(\beta^*, \theta^*))$ is stable and $\|M(P, K(\beta^*, \theta^*))\|_{\hat{\Delta}} < 1$. ∎

Remark 3: The choice of controller interpolation scheme affects the complexity of the multicontroller. Say, for example, that the candidate controller orders are $n_1, \ldots, n_p$. An interpolation approach that schedules the controller gains with respect to $\theta(t)$ will result in a multicontroller order of $\max_i\{n_i\}$. The *output-blending* scheme $u = \sum_{i=1}^{p} \beta_i u_i$ results in a multicontroller order of $\sum_{i=1}^{p} n_i$. To analyze the Q-blending scheme shown in Fig. 4(a), let us assume that the orders of $\tilde{Y}$ and $\tilde{X}^{-1}$ are n_0. Then the order of the Q-blending multicontroller is $2(n + n_0) + nn_0 \sum_{i=1}^{p} n_i$. The *a priori* theoretical properties of the Q-blending approach should be carefully weighed against its complexity.

B. Robust Adaptive Supervisor

The robust adaptive supervisor, shown in Fig. 1(b), is a dynamical system that takes as input the measured plant signals $u(t), y_m(t)$ and outputs the mixing signals $\beta(t)$ that "configures" the multicontroller $\mathbb{C}(\beta, \theta)$. Note that, because we are implementing the supervisor, the multicontroller may access the states of the supervisor, including the parameter estimate $\theta(t)$.

The mixer M implements the mapping $\beta : \Omega \rightarrow [0,\ 1]^p$. The following properties of M are assumed

M1. $\beta(\theta)$ is continuously differentiable.

M2. $\beta(\theta) \in \mathcal{B}_\theta, \quad \forall \theta \in \Omega$

Property M1, together with C1 ensures that if θ is tuned slowly then the closed-loop system will vary slowly. M2, together with C3, ensures that $\mathbb{C}(\beta(\theta), \theta)$ is a certainty equivalence stabilizing controller, i.e., for any $\theta^* \in \Omega$ the controller $\mathbb{C}(\beta(\theta^*), \theta^*)$ meets the control objective. Thus, if the control law $u = -K(s; \theta^*) y_m$ is applied to the nominal plant (14), the closed-loop system is internally stable, and the closed-loop characteristic polynomial $N_K(s; \theta^*) N_0(s; \theta_b^*) + D_K(s; \theta^*) D_0(s; \theta_a^*)$ is Hurwitz. Furthermore, when this control law is applied to the overall plant (13), the closed-loop system is stable if

$$\|\Delta_m\|_\infty \leq \min_{\theta^* \in \Omega} \left\| \frac{K(s; \theta^*) G(s; \theta^*)}{1 + K(s; \theta^*) G(s; \theta^*)} \right\|_\infty^{-1}. \tag{29}$$

If $\mathbb{C}(\beta, \theta)$ satisfies C3, the design of the mixer M is independent of $\mathbb{C}(\beta, \theta)$; otherwise, one must ensure, for all $\theta^* \in \Omega$, that $\mathbb{C}(\beta(\theta^*), \theta^*)$ meets the control objectives. Assuming requirement C3 is satisfied, the designer has considerable freedom in constructing M, and one such approach, as in Section III and in the sequal, is to define β based on the smooth bump function ψ given in Section II.

Because θ^* is unknown, $\mathbb{C}(\beta(\theta^*), \theta^*)$ cannot be calculated and, therefore, cannot be implemented. Thus, the adaptive mixing control approach replaces θ^* with its estimate θ. The well known counter example of Rohrs *et al.*, [38] demonstrates that for even small modeling errors that an adaptive system may become unstable. Thus, because of the presence of multiplicative uncertainty Δ_M, disturbance d, and sensor noise ν, we use a robust online parameter estimator to regain as much of the robustness of the known case as possible.

The robust parameter estimator comprises an error model $\mathbb{E}(\theta)$ and a robust adaptive law Σ_T, also referred to as the tuner. The error model $\mathbb{E}(\theta)$ is constructed by selecting an appropriate parameterization of the plant model. We proceed with the design of the error model by constructing a LPM using the same technique as in Section III. The interested reader is referred to [5, Sec. 2.4.1] for a detailed description. The LPM of the nominal system (14) is given by $z = \theta^{*T} \phi$, where

$$z = s^n F(s) y_m \tag{30}$$

$$\phi = [\,\alpha_{n-1}^T(s) F(s) u \quad -\alpha_{n-1}^T(s) F(s) y_m\,]^T \tag{31}$$

$$F = \frac{\lambda^n}{(s+\lambda)^n} F_\eta(s), \quad F_\eta(s) = \frac{N_F(s)}{D_F(s)} \tag{32}$$

where $\lambda > 0$ is a design constant and $F_\eta(s)$ is a proper stable minimum-phase filter. The estimation error ϵ_1 is generated by regarding $\theta(t)$ as the true parameter θ^*, i.e., $\epsilon_1 = z - \hat{z} = z - \theta^T \phi$. We define the *error model* $\mathbb{E}(\theta)$ as the dynamical system whose inputs are the observed data y_m and u, and its output is ϵ_1. The error model $\mathbb{E}(\theta)$ is realized from (30)–(32) and has the form

$$\dot{x}_\mathbb{E} = A_\mathbb{E} x_\mathbb{E} + B_\mathbb{E} u + G_\mathbb{E} y_m \tag{33}$$

$$z = C_z x_\mathbb{E} + D_z y_m, \quad \hat{z} = \theta^T C_\mathbb{E} x_\mathbb{E}, \quad \phi = C_\mathbb{E} x_\mathbb{E} \tag{34}$$

$$\epsilon_1 = (C_z - \theta^T C_\mathbb{E}) x_\mathbb{E} + D_z y_m \tag{35}$$

where $A_\mathbb{E}$ is Hurwitz and $\theta : \mathbb{R}^+ \rightarrow \mathbb{R}^{2n}$ is tuned by Σ_T. Note that $\mathbb{E}(\theta)$ is affine in θ.

When the error model $\mathbb{E}(\theta)$ is connected to the true plant $G(s)$ in the presence of the multiplicative perturbation $\Delta_m(s)$ and bounded disturbance d and noise ν, z is given by $z = \theta^{*T} \phi + \eta$, where

$$\eta = N_0(s) \Delta_m(s) F(s) u + D_0(s) F(s)(d + \nu) \tag{36}$$

is the modeling error term and acts as an estimation disturbance. The filter $F(s)$ can be chosen to mitigate the deleterious effects of η on estimation. For this purpose, $F(s)$ will typically have a bandpass frequency response to filter out signal bias and disturbances at low frequencies and sensor noise and unmodeled dynamics at high frequencies. For simplicity, we choose $F(s)$ to be analytic in $\mathcal{R}e[s] \geq -\delta_0/2$.

The robust adaptive law Σ_T can be implemented by a wide-class of algorithms [5, Chapter 8.5] whose dynamics take the fairly general form

$$\dot{\theta} = f_\theta(\theta, x_a, n_d, \epsilon_1, x_\mathbb{E}) \tag{37}$$

$$\dot{x}_a = f_a(x_a, \theta, n_d, x_\mathbb{E}) \tag{38}$$

$$\dot{n}_d = -\delta_0 n_d + |u|^2 + |y_m|^2 \tag{39}$$

where $x_T = [\theta^T\ x_a^T\ n_d]^T$ is the adaptive law's state; $x_a(t) \in \mathbb{R}^{n_a}$ is an auxiliary state required by the tuning algorithm; and n_d is the dynamic component of the normalization signal $m^2 = 1 + n_d$ that guarantees that $\phi/m, \eta/m \in \mathcal{L}_\infty$. We choose f_θ and f_a to be implemented with projection modification [4], [5] in order to constrain $\theta(t)$ to Ω. The adaptive law Σ_T is implemented by any of the algorithms found in [4], [5] with projection modification that guarantees

E1. $\epsilon, \epsilon m, \dot{\theta} \in \mathcal{S}(\eta^2/m^2) \cap \mathcal{L}_\infty$, if $\Delta_m \neq 0$, $d_0 \neq 0$, or $\nu_0 \neq 0$

E2. $\epsilon, \epsilon m, \dot{\theta} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, if $\Delta_m, d_0, \nu_0 = 0$

E3. $\theta \in \Omega,\ x_a \in \mathcal{L}_\infty$

where $\epsilon \triangleq \epsilon_1/m^2$ is the *normalized* estimation error. Additionally, we assume that the robust adaptive law satisfies

E4. $f_\theta(\cdot,\cdot,\cdot,0,\cdot) = 0$

i.e., adaptation ceases when $\epsilon_1 = 0$. This assumption is satisfied by all adaptive laws of [4], [5] except those with σ-modification.

Remark 4: If the plant parameters enter the model in a nonlinear fashion, one can often *overparameterize* the plant model. Overparameterization increases the class of admissible plants; thus, there may be some loss of performance, and, as in the example of [6, Sec. X], plant models that are not stabilizable may be introduced. Because the controller gains are computed off-line, adaptive mixing control avoids the computational and existence issues that arise in conventional adaptive control when stabilizability is lost. Furthermore, the adaptive law can be modified to ensure that $\theta(t)$ does not remain in a specified neighborhood about the points that lead to a loss of stabilizability (cf. 7.6 of [5]).

Remark 5: We believe that there will be no significant difficulties in extending adaptive mixing control to the MIMO case, which is currently under investigation.

C. Stability and Robustness Results

We now summarize the main results.

Theorem 4: Let the unknown plant be given by (13) and satisfying the plant assumptions P1-P5. Consider the adaptive mixing controller with the multicontroller $\mathbb{C}(\beta,\theta)$ given by (18) and satisfying assumptions C1-C3; error model given by (33)–(35); robust adaptive law Σ_T given by (37)–(39) and satisfying assumptions E1-E4; and mixer M satisfying M1-M2.

1) If $\Delta_m, d, \nu = 0$ then $x, \dot{x} \to 0$ as $t \to \infty$, where $x \triangleq [x_{\mathbb{P}}^T\ x_{\mathbb{C}}^T\ x_{\mathbb{E}}^T]^T$. Furthermore, let the multicontroller be given by the Q-blending scheme (25), and for some generalized plant P and $i = 1,\ldots,p$ let the candidate controller K_i yield robust performance for all $\theta^* \in \Omega_i$. If $\lim_{t\to\infty}\theta(t) = \theta^*$, then $x_{\mathbb{PC}} \triangleq \left[x_{\mathbb{P}}^T\ x_{\mathbb{C}}^T\right]^T \to x_{\mathbb{PC}}^*$ as $t \to \infty$, where $x_{\mathbb{PC}}^*(t)$ is the solution to
$$\dot{x}_{\mathbb{PC}}^* = \begin{bmatrix} A_{\mathbb{P}} & -B_{\mathbb{P}}C_{\mathbb{C}}^* \\ B_{\mathbb{C}}^*C_{\mathbb{P}} & A_{\mathbb{C}}^* \end{bmatrix} x_{\mathbb{PC}}^* = A_{\mathbb{PC}}^* x_{\mathbb{PC}}^* \tag{40}$$
and the triplet $(A_{\mathbb{C}}^*,\ B_{\mathbb{C}}^*,\ C_{\mathbb{C}}^*)$ is the state-space realization of some controller that yields robust performance with respect to the generalized plant P.

2) There exists $\delta^* > 0$ such that if $c\Delta_1^2 < \delta^*$, where $\Delta_1 \triangleq \|N_0\Delta_m F\|_{2\delta_0}$ and $c > 0$ is a finite constant, then the adaptive mixing control scheme guarantees that $x, \dot{x}, \theta, \dot{\theta}, x_T \in \mathcal{L}_\infty$. Furthermore, there exist constants $c_0, c_1 > 0$ such that
$$\int_0^t |y(\tau)|^2 d\tau \le c_0\mu^2 t + c_1 \tag{41}$$
where $\mu^2 = c(\Delta_1^2 + d_0^2 + \nu_0^2)$. □

The proof is given in the Appendix . In result 1), exponential convergence of estimate θ to the true value θ^* can be guaranteed provided ϕ is *persistently exciting* (PE). The control input can be augmented with an auxiliary signal $u_a(t)$ that forces u to be *sufficiently rich of order* $2n$, guaranteeing ϕ is PE if the plant is controllable and observable. For output tracking of controllable, observable plants, the reference signal r may naturally ensure that ϕ is PE; otherwise, guaranteeing that ϕ is PE can be accomplished by augmenting r or u to be sufficiently rich of order $2n$. The interested reader is referred to Chapters 3 and 4 of [5].

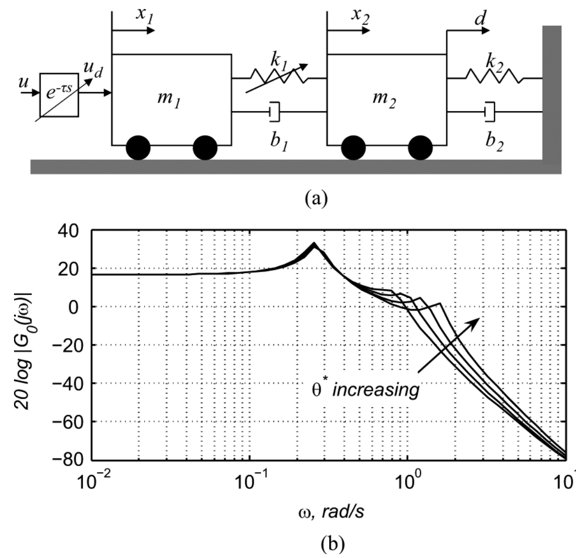

Fig. 5. Mass-spring-dashpot system benchmark example (a) The two-cart system possesses an unmodeled control delay, uncertain spring constant, process disturbance, and measurement noise (b) Bode plot of nominal model for various values of $\theta^* = k_1$.

V. A Benchmark Example

In this section, we consider the two-cart mass-spring-dashpot (MSD) system [19] shown in Fig. 5(a). The parameters $m_1 = m_2 = 1$, $k_2 = 0.15$, and $b_1 = b_2 = 0.1$ are known, while $\theta^* \triangleq k_1$ is known to belong to $\Omega = \{k : 0.25 \le k \le 1.75\}$. The plant disturbance $d(t)$ is a low-frequency, stationary stochastic process that acts on m_2 and is generated by $d = a/(s + a\xi)$, where $a = 0.1$ is the disturbance bandwidth and the white gaussian process $\xi(t)$ has zero mean and unit intensity ($\Xi = 1$). The measurement is m_2's displacement plus additive white gaussian noise, i.e., $y_m(t) = x_2(t) + \nu(t)$, where $E\{\nu(t)\} = 0$ and $E\{\nu(t)\nu(\tau)\} = 10^{-6}\delta(t-\tau)$.

The control $u(t)$ is applied to m_1 through a control channel with an maximum time delay of 0.05 s. The plant output is given by
$$y_m = G_0(s)(1 + \Delta_m(s))u + G_\xi(s)\xi + \nu$$
$$G_0(s) = \frac{N_0(s)}{D_0(s)} = \frac{N_0^K(s) + \theta^*}{D_0^K(s) + \theta^* D_0^U(s)}$$
where $\Delta_m = e^{-\tau s} - 1$ is the multiplicative unmodeled dynamics due to the time delay; $G_\xi(s)$ is the transfer function from disturbance to plant output; $N_0^K(s) \triangleq 0.1s$; $D_0^K(s) \triangleq s^4 + 0.3s^3 + 0.16s^2 + 0.015s$; and $D_0^U(s) \triangleq 2s^2 + 0.1s + 0.15$. The performance variable $z(t) = x_2(t)$ is to be kept small.

In [19], an RMMAC scheme was developed. First, the candidate controller set $\mathcal{C} = \{K_i(s)\}_{i=1}^4$, where each $K(s) \in \mathcal{C}$ is a mixed-μ compensator, and the corresponding parameter partition $\tilde{\Omega}_1 = [1.02,\ 1.75]$, $\tilde{\Omega}_2 = [0.64,\ 1.02]$, $\tilde{\Omega}_3 = [0.40,\ 0.64]$, and $\tilde{\Omega}_4 = [0.25,\ 0.40]$ were generated by the $\%FNARC$ method (cf. [19].) The supervisor was constructed using the RMMAC approach. The control is generated by weighting each controller output by the output of the supervisor, i.e., $u(t) = \sum_{i=1}^{4} \beta_i(t) u_i(t)$.

For a fair comparison, the adaptive mixing control and supervisory control schemes utilize the candidate controller set $\mathcal{C}$ developed in the RMMAC design. Let us now consider the design of an adaptive mixing control scheme. We start the design of the multicontroller $K(\beta)$ by enlarging each parameter subset $\tilde{\Omega}$ so that mixing regions are artificially introduced. After expanding the boundaries of $\tilde{\Omega}_i, i = 1, \ldots, 4$, by 10%, we have the new parameter partition $\Omega_1 = [0.92,\ 1.93]$, $\Omega_2 = [0.58,\ 1.12]$, $\Omega_3 = [0.36,\ 0.70]$, and $\Omega_4 = [0.23,\ 0.44]$. The multicontroller design is completed by performing output blending, i.e., $u(t) = \sum_{i=1}^{4} \beta_i(t) u_i(t)$. It is clear that requirements C1 and C2 are satisfied. Using standard mixed-μ analysis tools, we have found property C3 holds over Ω. Also, for comparison, an adaptive mixing control scheme was developed using the Q-blending approach to construct the multicontroller.

The design of the mixing system M is accomplished by defining the functions $\tilde{\beta}_i, i = 1, \ldots, 4$, as

$$\tilde{\beta}_1(\theta) = \psi\left(\frac{\theta - 1.422}{0.504}\right), \quad \tilde{\beta}_2(\theta) = \psi\left(\frac{\theta - 0.849}{0.273}\right)$$
$$\tilde{\beta}_3(\theta) = \psi\left(\frac{\theta - 0.532}{0.172}\right), \quad \tilde{\beta}_4(\theta) = \psi\left(\frac{\theta - 0.333}{0.108}\right)$$

where ψ is the smooth bump function. The mixing signal β is generated by normalizing $\tilde{\beta} = [\tilde{\beta}_1 \ \cdots \ \tilde{\beta}_4]^T$, i.e., $\beta = \tilde{\beta} / \sum_{i=1}^{4} \tilde{\beta}_i$, and requirements M1 and M2 are satisfied.

The final component of the adaptive mixing control scheme is the robust adaptive law. The LPM $z(t) = \theta^* \phi(t) + \eta(t)$ is used to derive the adaptive law, where

$$z(t) = D_0^K(s) F(s) y_m(t) - N_0^K(s) F(s) u(t) \tag{42}$$
$$\phi(t) = F(s) u(t) - D_0^U(s) F(s) y_m(t), \tag{43}$$
$$F(s) = \frac{5^4}{(s+5)^4} F_\eta(s). \tag{44}$$

$\eta(t)$ is the modeling error; $F_\eta(s)$ is the bandpass filter

$$F_\eta = k_{bp} \frac{(s^2 + \epsilon_{bp} s + 0.005688)(s^2 + \epsilon_{bp} s + 0.02807)}{(s^2 + 0.08s + 0.08)(s^2 + 0.543s + 0.178)} \cdot \frac{(s^2 + \epsilon_{bp} s + 80.15)(s^2 + \epsilon_{bp} s + 395.6)}{(s^2 + 4.59s + 12.67)(s^2 + 1.5s + 28.09)}$$

where $k_{bp} = 0.010001$ and $\epsilon_{bp} = 1 \times 10^{-5}$. The passband is $W_P = [.3,\ 5]$ rad/s, with a peak-to-peak 0.25 dB ripple; and the stopband is $W_S = [.1,\ 10]$ rad/s, with a -40 dB attenuation level. These values were chosen to make use of the frequency range that is largely affected by θ^*, as shown graphically in Fig. 5(b), while filtering out the remaining frequencies that may become dominated by η.

An adaptive law using the gradient method based on integral cost is used:

$$\dot{\theta} = \begin{cases} -5\,(r\theta + q), & \text{if } |\theta - 1| < 0.75 \text{ or} \\ & \text{if } 5\,(r\theta + q)\,\mathrm{sgn}(\theta) \geq 0 \\ 0, & \text{otherwise} \end{cases}$$
$$\dot{r} = -0.1r + \frac{\phi^2}{m^2}, \quad \dot{q} = -0.1q - \frac{z\phi}{m^2},$$
$$r(0) = q(0) = 0$$

TABLE I
MODEL ASSUMPTIONS SATISFIED

	θ^*	Short-term RMS	Long-term RMS
AMC		0.021	0.012
RMMAC	1.75	0.014	0.012
SAC		0.016	0.010
AMC		0.018	0.012
RMMAC	1.385	0.016	0.012
SAC		0.023	0.012
AMC		0.016	0.016
RMMAC	1.02	0.022	0.020
SAC		0.019	0.019
AMC		0.014	0.013
RMMAC	0.830	0.015	0.013
SAC		0.023	0.012
AMC		0.018	0.017
RMMAC	0.640	0.019	0.017
SAC		0.023	0.017
AMC		0.015	0.014
RMMAC	0.520	0.014	0.013
SAC		0.018	0.014
AMC		0.019	0.018
RMMAC	0.400	0.017	0.018
SAC		0.020	0.021
AMC		0.018	0.017
RMMAC	0.325	0.015	0.014
SAC		0.018	0.014
AMC		0.019	0.017
RMMAC	0.25	0.016	0.017
SAC		0.018	0.016

where $\theta(t)$ is the estimate of θ^* and $m^2(t) \triangleq 1 + n_d$ is the dynamic normalization signal, where

$$\dot{n}_d = -0.04 n_d + (F_\eta(s) u)^2 + (F_\eta(s) y)^2, \quad n_d(0) = 0.$$

The design parameters of the adaptive law were chosen by trial and error.

The supervisory adaptive control scheme (labeled as SAC in Table I and the figures) is constructed by monitoring the estimation error signals $\epsilon_i = z(t) - \theta_i^* \phi(t)$ for $i = 1, 2, 3, 4$, where each constant θ_i^* is chosen as the center of the interval $\tilde{\Omega}_i$, and $z(t)$ and $\phi(t)$ are generated by (42) and (43), respectively. Note that the filters that generate $z(t)$ and $\phi(t)$ are identical to the filters in the adaptive mixing control scheme. Thus, any effect attributed to the filter $F(s)$ is experienced by both schemes. The supervisory scheme generates the monitoring signals $\mu_i = \int_0^t e^{-0.04(t-\tau)} \epsilon_i(\tau) d\tau$ $(i = 1, 2, 3, 4)$, and selects the controller corresponding to the smallest monitoring signal subject to a hysteresis logic. Hysteresis logic prohibits switching unless the smallest monitoring signal is at least 0.1% smaller than the currently selected monitoring signal. The value of the hysteresis constant was chosen by trial and error based on an acceptable compromise between rapid response and reduced likelihood of switching due to noise and modeling error.

Fig. 6 shows the results of two different adaptive mixing controllers: an output blending scheme and a Q-blending scheme. The output-blending scheme exhibits better transient and long-term performance compared to the Q-blending mixing scheme, which is more susceptible to bursting-type behaviors because the Q-blending multicontroller depends on both $\beta(t)$ and $\theta(t)$. This makes it more susceptible to θ variations.

Based on the above results, we chose to use the adaptive mixing control scheme with output blending to compare with the RMMAC and supervisory adaptive control schemes. Although an extensive evaluation of the adaptive mixing control,

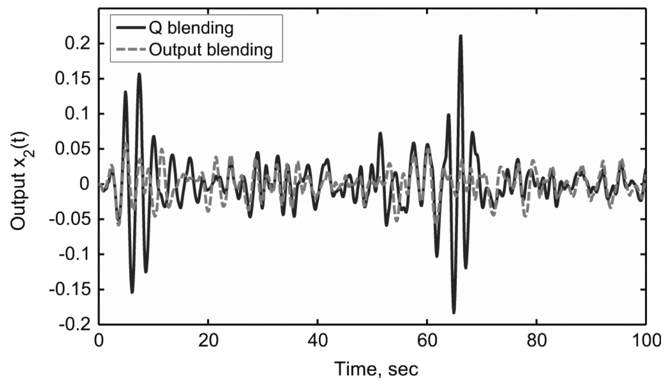

Fig. 6. Comparison of the plant outputs of the Q-blending and output-blending schemes with $\theta^* = 0.325$, $\tau = 0.05$, and nominal disturbance model.

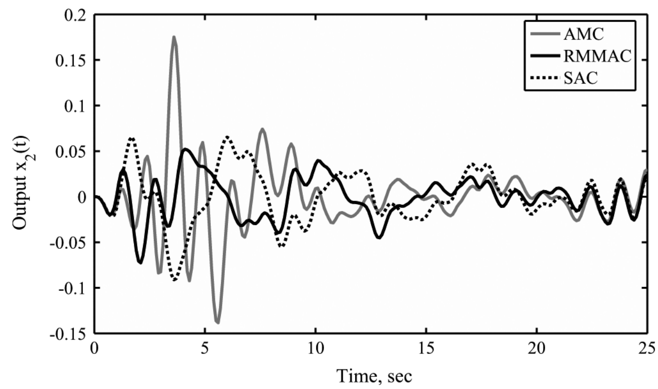

Fig. 7. Comparison of the start-up transients of adaptive mixing control (AMC), RMMAC, and supervisory adaptive control (SAC): $\theta^* = 1.75$, $\tau = 0.05$, and nominal disturbance model.

RMMAC, and supervisory schemes are beyond the scope of this paper[3], we present some simulation results that illustrate some of the potential benefits of the adaptive mixing control scheme. Table I presents short-term $(0 \leq t \leq 100)$ and long-term $(100 \leq t \leq 200)$ output RMS values for various values of θ^*, and with $\tau = 0.05$ and all model assumptions satisfied. The results of Table I are the average of five trials.

While the three schemes achieve remarkable performance, the following observations were made. In general, the RMMAC scheme exhibits the smallest start-up transient if θ^* takes on values near the boundary of Ω (see Fig. 7), whereas the adaptive mixing controller typically has the smallest if θ^* takes on values around 1 (see Fig. 8). The supervisory adaptive scheme tends to perform a number a switches before reliable data are observed, often creating a large transient (see Fig. 8). These observations are congruent with the short-term RMS values of Table I. Additionally, when θ^* takes on a value near the boundary of two candidate models, the supervisory adaptive controller may create additional transients by switching between controllers. This last observation is illustrated in Fig. 8, particularly Figs. 8(c) and 8(d). For a clear presentation, we have plotted the results pairwise, although the schemes were simulated side-by-side.

Next we consider the off-nominal case in which the disturbance model's power and bandwidth are increased to 100 and 3, respectively, and all control designs remain unchanged. The results are shown in Fig. 9. Because the increased disturbance power and bandwidth, the magnitudes of the residual signals of the RMMAC's supervisor are larger than expected[4] and causes the degraded performance[5] seen in Fig. 9(a) and (b). The adaptive mixing control and supervisory adaptive schemes, however, are reasonably robust to this plant disturbance, as illustrated in Fig. 9(c) and (d).

[3]Due to space limitations, we have focused on the constant θ^* case. We would like to remark that the mixing scheme has performed well for slow time-variations, say for example $\theta^*(t) = 1 - 0.75\sin(0.01t)$. To cope with the faster time variation of $\theta^*(t) = 1 - 0.75\sin(0.05t)$, we made the changes $\delta_0 = 0.4$ and $\beta = 1$ to achieve satisfactory performance, but at the cost of slight degradation of nominal performance.

[4]The residual signals become so large that numerical round-off causes divide-by-zero conditions in the RMMAC algorithm. Here, this condition is handled by outputting the supervisor's previous output. This approach has maintained reasonable levels of the plant output in our simulations.

[5]RMMAC's performance can likely be regained by extending the RMMAC/XI scheme to cope with a bounded range of a in addition to a range of Ξ. This comes at the cost of additional complexity.

VI. Conclusion

This paper presents the adaptive mixing control approach. The motivation for adaptive mixing control is to develop a deterministic approach that is capable of achieving high-performance by utilizing well established robust LTI tools, while avoiding issues of undesirable switching behaviors and uncertain disturbance models. For the nominal case, it has been shown that the adaptive mixing control scheme drives the plant states to zero. We have also shown that when the plant parameter estimates converge to the their true values, which can be guaranteed by a persistence of excitation condition, the control objective, in terms of a LTI robust performance specification, is met exponentially. In the presence of unmodeled dynamics, noises, and disturbances, the regulation error is of the order of these modeling uncertainties in the mean square sense. The 2-cart MSD example has demonstrated that adaptive mixing control can yield satisfactory robustness to perturbations in the disturbance model, while avoiding some of the poor behaviors associated with undesirable switching. These promising results warrant further evaluations.

Because discontinuous switching logic has been replaced with smooth, stable controller interpolation, adaptive mixing control schemes may not be able to respond to dramatic changes in the plant as rapidly as switching-based adaptive control schemes can. This is a disadvantage of the proposed scheme. Thus, a direction of future research is to develop adaptive control schemes that retain the desirable run-time properties of the mixing and switching schemes. The aim would be to have the supervisor switch in response to large parameter changes and then mix in response to subtle changes in the estimated model.

Appendix

Proof of Theorem 4: Let us define the *parameterized controller* $\Sigma_C(\theta) \triangleq \mathbb{C}(\beta(\theta), \theta)$ as the mixer-multicontroller interconnection. For any constant $\theta \in \Omega$, $\Sigma_C(\theta)$ has the transfer function (20). We also define the *parameterized system* $\Sigma(\theta)$ as the plant-parameterized controller-error model interconnection, with its output chosen as ϵ_1. From (16), (18), (33)–(35), the parameterized system $\Sigma(\theta)$ can be written compactly as

$$\dot{x} = A(\theta)x + B(d+\nu), \quad \epsilon_1 = C(\theta)x \tag{45}$$

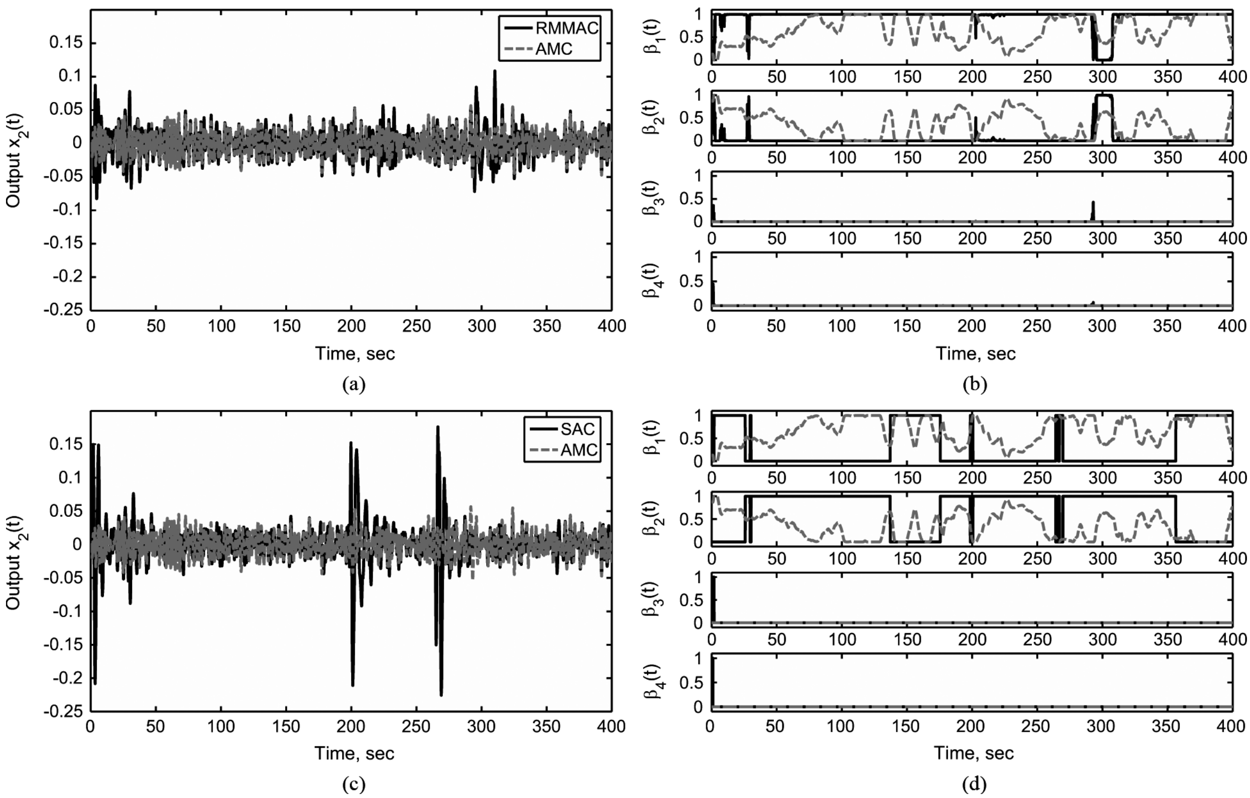

Fig. 8. Simulation results with $\theta^* = 1.02$, $\tau = 0.05$, and nominal disturbance model: Adaptive mixing control (AMC), RMMAC, and supervisory adaptive control (SAC) (a) Plant output: AMC and RMMAC (b) Controller weights: AMC and RMMAC (c) Plant output: AMC and SAC (d) Controller weights: AMC and SAC.

where $x \triangleq [x_{\mathbb{P}}^T \, x_{\mathbb{C}}^T \, x_{\mathbb{E}}^T]^T$ and the triplet $(C(\theta),\ A(\theta),\ B)$ is defined in the obvious manner. The closed-loop adaptive system $(\Sigma(\theta(t)), \Sigma_T)$ is formed by replacing the parameter θ of the parameterized system Σ with the tuned estimates $\theta(t)$ generated by the robust adaptive law Σ_T. The closed-loop system (45) is in a form suitable for analysis using the tunability approach of [39]. Also, it has been establish in [40] that along the trajectories of $(\Sigma(\theta(t)), \Sigma_T)$ there exists a unique global solution $[x^T(t)\ x_T^T(t)]^T, \forall t \in [0, \infty)$.

Step 1: Establish that for all fixed $\theta \in \Omega$, $\{C(\theta),\ A(\theta)\}$ is a detectable pair.

Consider the adaptive law initialization $\theta(0) = \theta_0 \in \Omega$, where $\theta_0 = [\theta_{b_0}^T\ \theta_{a_0}^T]^T$ is any fixed constant in Ω. The vectors $\theta_{b_0}^T$, $\theta_{a_0}^T$ represent the initial estimates of the coefficients of the $N_0(s)$ and $D_0(s)$, respectively. Let $d \equiv 0$, $\nu \equiv 0$, and $\epsilon_1 \equiv 0$. Thus, it follows that $y_m \equiv y$ and, because $\epsilon_1 \equiv 0$, there is no adaptation, i.e., $\theta \equiv \theta_0$. Therefore the closed-loop system is an LTI system. Since $\epsilon_1 \equiv 0$, we have $z = \theta_0^T \phi$. Thus, it follows from (30) and (31) that the signals y_m and u satisfy

$$F(s)\underbrace{(s^n + \theta_{a_0}^T \alpha_{n-1}(s))}_{D_0(s;\theta_{a_0})} y_m = F(s)\underbrace{\theta_{b_0}^T \alpha_{n-1}(s)}_{N_0(s;\theta_{b_0})} u. \tag{46}$$

Similarly, the parameterized controller $\Sigma_{\mathbb{C}}(\theta_0)$ is an LTI system and y_m and u also satisfy

$$u = -K(s;\theta_0) y_m = -\frac{N_K(s;\theta_0)}{D_K(s;\theta_0)} y_m. \tag{47}$$

From (46) and (47), y_m and u satisfy

$$\begin{bmatrix} F(s)D_0(s;\theta_{a_0}) & -F(s)N_0(s;\theta_{b_0}) \\ N_K(s;\theta_0) & D_K(s;\theta_0) \end{bmatrix} \begin{bmatrix} y_m \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The characteristic equation of the above system is

$$F(s)(D_0(s;\theta_{a_0})D_K(s;\theta_0) + N_0(s;\theta_{b_0})N_K(s;\theta_0)) = 0.$$

Since $F(s)$ is a stable minimum phase filter and $D_0(s;\theta_{a_0})D_K(s;\theta_0) + N_0(s;\theta_{b_0})N_K(s;\theta_0)$ is Hurwitz by properties C3 and M2, we have that $y_m, u \to 0$ as $t \to \infty$. From the detectability of $(C_{\mathbb{P}},\ A_{\mathbb{P}})$ and $(C_{\mathbb{C}}(\beta(\theta_0), \theta_0),\ A_{\mathbb{C}}(\beta(\theta_0), \theta_0))$, together with the convergence of y_m and u to zero, it follows that $x_{\mathbb{P}}, x_{\mathbb{C}} \to 0$ as $t \to \infty$. Since $A_{\mathbb{E}}$ is a stability matrix, the convergence of y_m and u to zero implies that $x_{\mathbb{E}} \to 0$ as $t \to \infty$. Therefore, because it has been shown that x converges to zero for all $\theta_0 \in \Omega$ when $d, n, \epsilon_1 \equiv 0$, the parameterized pair $(C(\theta_0),\ A(\theta_0))$ is detectable on Ω.

Step 2: Establish that along the solutions of $(\Sigma(\theta),\ \Sigma_t)$ there exists a function $L : \Omega \to \mathbb{R}^{1\times \bar{n}}$ such that $A_I(t) \triangleq A(\theta(t)) - L(\theta(t))C(\theta(t))$ is exponentially stable.

Recall that the robust adaptive law guarantees that $\epsilon, \epsilon m, \dot{\theta} \in \mathcal{S}(\eta^2/m^2)$. Applying [5, Lemma 3.3.2] to (36), together with $|d| \le d_0$ and $|\nu| \le \nu_0$, yields

$$|\eta(t)| \le \Delta_1 \|u_t\|_{2\delta_0} + \Delta_2 \tag{48}$$

$$\Delta_1 \triangleq \|N_0 F \Delta_m\|_{2\delta_0},$$

$$\Delta_2 \triangleq \|D_0 F\|_{\infty - gn}(d_0 + \nu_0) \tag{49}$$

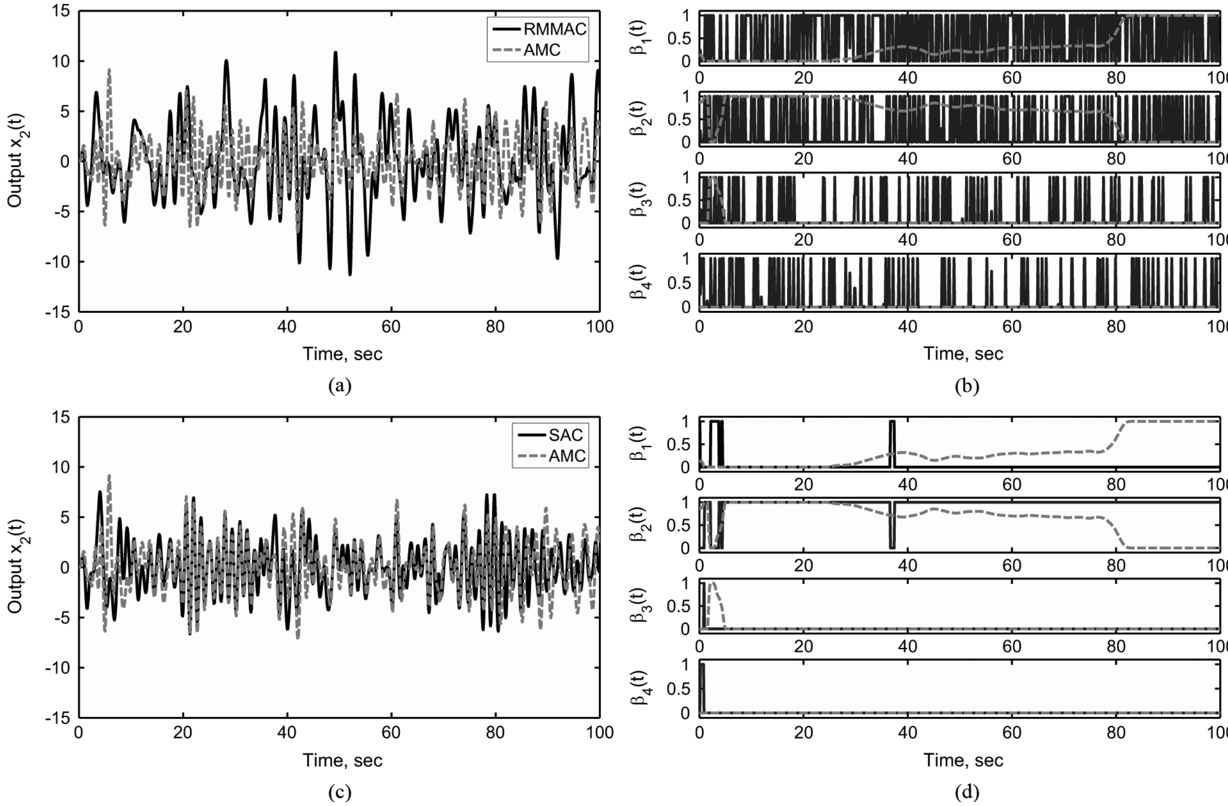

Fig. 9. Simulation results with $\theta^* = 1.02$, $\tau = 0.05$, and the off-nominal disturbance model with a disturbance power of $100 \cdot \Xi$ and a bandwidth of $30 \cdot a$: Adaptive mixing control (AMC), RMMAC, and supervisory adaptive control (SAC) (a) Plant output: AMC and RMMAC (b) Controller weights: AMC and RMMAC (c) Plant output: AMC and SAC (d) Controller weights: AMC and SAC.

and since $m^2 = 1+\|u_t\|^2_{2\delta_0}+\|(y_m)_t\|^2_{2\delta_0}$ and $m \geq 1$, it follows that $|\eta(t)|/m \leq \Delta_1 + \Delta_2$. Therefore, ϵ, ϵm, $\dot{\theta} \in \mathcal{S}(\mu^2)$, where $\mu^2 \triangleq c(\Delta_1^2 + \Delta_2^2)$ and c is some constant.

Because properties C1 and M1 guarantee that $\mathbb{C}(\beta(\theta), \theta)$ and $\beta(\theta)$, respectively, are continuously differentiable, the parameterized controller Σ_C is continuously differentiable with respect to θ. The error model $\mathbb{E}(\theta)$ is affine in θ and, therefore, continuously differentiable. Consequently, the pair $(C(\theta),\ A(\theta))$ is continuously differentiable with respect to θ. Furthermore, because the adaptive law guarantees that $\theta(t) \in \Omega$ and $\dot{\theta} \in \mathcal{S}(\mu^2)$, it follows from the detectability result of Step 1 and result 2) of Theorem 1 that there exists a continuously differentiable function $L : \Omega \rightarrow \mathbb{R}^{\bar{n}\times 1}$ such that $A_I(t) \triangleq A(\theta(t)) - L(\theta(t))C(\theta(t))$ is e.s. provided that $\mu^2 < \mu^*$ for some μ^*. For large Δ_2, i.e., large d_0 or ν_0, this condition may not be satisfied even for small Δ_1. By using the lengthy analysis approach of [5, Section 9.9.1], involving a contradiction argument, boundedness of the closed-loop signals can be proven provided Δ_1 satisfies a bound condition that is independent of Δ_2. However, for simplicity, we continue with an alternative analysis approach, where we assume the filter $F(s)$ is chosen so that Δ_2 is sufficiently small, say $c\Delta_2^2 < \mu^*/2$, so that for $c\Delta_1^2 < \mu^*/2$, the inequality $\mu^2 < \mu^*$ is always satisfied. Therefore, $A_I(t)$ is e.s., i.e., the transition matrix $\Phi(t,\tau)$ of $A_I(t)$ satisfies $\|\Phi(t,\tau)\| \leq \lambda_0 e^{-\alpha_0(t-\tau)}$ for some positive constants λ_0, α_0 and $t \geq \tau \geq 0$. Note that if $\Delta_m, d, \nu = 0$, the adaptive law guarantees that $\dot{\theta} \in \mathcal{L}_2$, and from result 1) of Theorem 1, $A_I(t)$ is e.s. Since $L(t)$ is continuous and Ω is compact, $\|L\| \in \mathcal{L}_\infty$, where $L(t)$ is a slight abuse of notation and is taken to mean $L(\theta(t))$.

Step 3: Establish boundedness and convergence of x

Let $\delta \in [0, \delta_1)$, where $\delta_1 < \min\{2\alpha_0, \delta_0\}$, and $c > 0$ denotes any finite constant. Recall that δ_0 is defined in (39).

By applying output injection, we rewrite (45) as

$$\dot{x} = A_I(t)x + B(d+\nu) + L(t)\epsilon_1 \tag{50}$$

where in Step 2 we established e.s. of the homogeneous part of (50).

We establish that $m \in \mathcal{L}_\infty$: By Lemma 3.3.3 of [5] and the e.s. property of A_I, we have that

$$\|x_t\|_{2\delta} \leq c\|\,(\epsilon_1)_t\,\|_{2\delta} + c \tag{51}$$

where $\epsilon_1 \in \mathcal{L}_{2e}$ because ϵ_1 is a continuous function of time. Applying the $\mathcal{L}_{2\delta}$ norm to $y_m = C_{\mathbb{P}}x_{\mathbb{P}} + d + \nu$ and $u = -C_{\mathbb{C}}(t)x_{\mathbb{C}}$, where $\|C_{\mathbb{C}}(t)\|$ is bounded (a consequence of the continuously differentiability of $C_{\mathbb{C}}(\beta,\ \beta(\theta))$ and the compactness of Ω), yields

$$\|(y_m)_t\|_{2\delta} \leq c\|(x_{\mathbb{P}})_t\|_{2\delta} + c \leq c\|(\epsilon_1)_t\|_{2\delta} + c \tag{52}$$

$$\|u_t\|_{2\delta} \leq c\|(x_{\mathbb{C}})_t\|_{2\delta} \leq c\|(\epsilon_1)_t\|_{2\delta} + c \tag{53}$$

where the second inequalities of (52) and (53) were obtained by first recognizing that $x_{\mathbb{P}}$ and $x_{\mathbb{C}}$ are subvectors of x and then applying inequality (51). Consider the *fictitious normalization signal* $m_f^2 \triangleq 1+\|u_t\|^2_{2\delta}+\|(y_m)_t\|^2_{2\delta}$. Note that because $\delta < \delta_0$, it follows from the definitions of m and m_f that $m \leq m_f$.

Substituting (52), (53), and $\epsilon_1 = \epsilon m^2$ into the definition of m_f yields

$$m_f^2 \leq c\|(\epsilon m^2)_t\|_{2\delta}^2 + c \leq c\|(\epsilon m m_f)_t\|_{2\delta}^2 + c \tag{54}$$

where the second inequality is obtained by using $m \leq m_f$. From the definition of $\|(\cdot)_t\|_{2\delta}$ it follows that:

$$m_f^2 \leq c\int_0^t e^{-\delta(t-\tau)}(\epsilon(\tau)m(\tau))^2 m_f^2(\tau)d\tau + c. \tag{55}$$

Applying the Bellman-Gronwall Lemma (cf. [5, Lemma 3.3.9]) to (55) yields the inequality $m_f^2 \leq ce^{-\delta t}e^{c\int_0^t g^2(\tau)d\tau} + c\delta\int_0^t e^{-\delta(t-s)}e^{c\int_s^t g^2(\tau)d\tau}ds$, where $g = \epsilon m$. Let us assume that $F(s)$ is chosen such that $c\Delta_2^2 \leq \delta/2$. Because $\epsilon m \in \mathcal{S}(\mu^2)$ implies $c\int_s^t(\epsilon(\tau)m(\tau))^2 d\tau \leq c\mu^2(t-s)$, it follows that for $c\Delta_1^2 \leq \delta/2$, we have $m_f \in \mathcal{L}_\infty$. Since $m \leq m_f$, we have that $m \in \mathcal{L}_\infty$, and together with $\epsilon m \in \mathcal{S}(\mu^2)\cap\mathcal{L}_\infty$ (property E1) implies that $\epsilon_1 = \epsilon m^2 \in \mathcal{S}(\mu^2)\cap\mathcal{L}_\infty$. Moreover, it follows that $n_d \in \mathcal{L}_\infty$ because $m^2 = 1 + n_d$, which, together with property E3, implies $x_T = \left[\theta^T\ x_a^T\ n_d\right]^T \in \mathcal{L}_\infty$.

We now turn our attention to the injected system (50). Recall that $A_I(t)$ is e.s., d and ν are bounded, and $\bar{U} = L\epsilon_1$ is in $\mathcal{S}(\mu^2)\cap\mathcal{L}_\infty$. Therefore, $x \in \mathcal{L}_\infty$, and, in turn, $\dot{x} \in \mathcal{L}_\infty$. Now we examine the mean-square properties of x. From [5, Corollary 3.3.3] it follows that $x \in \mathcal{S}(\mu^2)$ and, in turn, $y \in \mathcal{S}(\mu^2)$ since y is a subvector of x. Thus, (41) holds. To summarize, the condition for stability is $c\Delta_1^2 < \delta^* \triangleq \min\{\mu^*/2, \delta/2\}$, for some constants $0 < \delta < \min\{\delta_0, 2\alpha_0\}$ and $c > 0$, where $\mu^* > 0$ is the bound for μ^2 such that $A_I(t)$ is e.s.

Let us consider that $\Delta_m, d, \nu \equiv 0$. For this case, $\bar{U} = L\epsilon_1$ is a $\mathcal{L}_2\cap\mathcal{L}_\infty$ function because $\|L\| \in \mathcal{L}_\infty$ and $\epsilon_1 \in \mathcal{L}_2\cap\mathcal{L}_\infty$ (from E2 and $m \in \mathcal{L}_\infty$). Thus, since $A_I(t)$ is e.s., we have $x \in \mathcal{L}_2\cap\mathcal{L}_\infty$, $\dot{x} \in \mathcal{L}_2\cap\mathcal{L}_\infty$, and $x \to 0$ as $t \to \infty$. From the convergence of x, and consequently $L\epsilon_1$, it follows from (50) that $\dot{x} \to 0$ as $t \to \infty$.

We now consider the case that $\lim_{t\to\infty}\theta(t) = \theta^*$. It follows from (16) and (18) that:

$$\dot{x}_{\mathbb{PC}} = \begin{bmatrix} A_{\mathbb{P}} & -B_{\mathbb{P}}C_{\mathbb{C}}(\beta(\theta),\theta) \\ B_{\mathbb{C}}(\beta(\theta),\theta)C_{\mathbb{P}} & A_{\mathbb{C}}\beta(\theta),\theta) \end{bmatrix} x_{\mathbb{PC}} \tag{56}$$

$$= A_{\mathbb{PC}}(t)x_{\mathbb{PC}} \tag{57}$$

where $x_{\mathbb{PC}} = [x_{\mathbb{P}}^T\ x_{\mathbb{C}}^T]^T$ and $\theta(t)$ is tuned by the adaptive law. Let $A_{\mathbb{PC}}^* \triangleq \lim_{t\to\infty} A_{\mathbb{PC}}(t)$. From Lemma 3, the triplet $(A_{\mathbb{PC}}^*,\ B_{\mathbb{PC}}^*,\ C_{\mathbb{PC}}^*)$ is the state space realization of a controller that yields robust performance for the generalized plant P. From (40) and (57), the dynamics of $\tilde{x} = x_{\mathbb{PC}} - x_{\mathbb{PC}}^*$ is given by $\dot{\tilde{x}} = A_{\mathbb{PC}}^*\tilde{x} + \tilde{u}$, $\tilde{u} \triangleq (A_{\mathbb{PC}}(t) - A_{\mathbb{PC}}^*)x_{\mathbb{PC}}$. Because $A_{\mathbb{PC}}^*$ is Hurwitz and $\lim_{t\to\infty}\tilde{u} \to 0$, we have $\lim_{t\to\infty}\tilde{x}(t) = 0$. Therefore, $x_{\mathbb{PC}} \to x_{\mathbb{PC}}^*$ as $t \to \infty$. ∎

REFERENCES

[1] P. M. Young, "Controller design with mixed uncertainties," in *Proc. Amer. Control Conf.*, Baltimore, MD, Jun. 1994, pp. 2333–2337.

[2] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*, 2nd ed. New York: Wiley-Interscience, 2005.

[3] K. Zhou and J. C. Doyle, *Essentials of Robust Control*. Englewood Cliffs, NJ: Prentice Hall, 1997.

[4] P. A. Ioannou and B. Fidan, *Adaptive Control Tutorial, ser. Advances in Design and Control*. Philadelphia, PA: SIAM, 2006.

[5] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Upper Saddle River, NJ: Prentice-Hall, 1996.

[6] A. S. Morse, "Supervisory control of linear set-point controllers—Part 1: Exact matching," *IEEE Trans. Autom. Control*, vol. 41, no. 10, pp. 1413–1431, Oct. 1996.

[7] A. S. Morse, "Supervisory control of linear set-point controllers—Part 2: Robustness," *IEEE Trans. Autom. Control*, vol. 42, no. 11, pp. 1500–1515, Nov. 1997.

[8] J. Hespanha, D. Liberzon, A. S. Morse, B. D. O. Anderson, T. S. Brinsmead, and F. De Bruyne, "Multiple model adaptive control. Part 2: Switching," *Int. J. Robust Nonlin. Control*, vol. 11, no. 5, pp. 479–496, Apr. 2001.

[9] K. S. Narendra and J. Balakrishnan, "Improving transient response of adaptive control systems using multiple models and switching," *IEEE Trans. Autom. Control*, vol. 39, no. 9, pp. 1861–1866, Sep. 1994.

[10] K. S. Narendra and C. Xiang, "Adaptive control of discrete-time systems using multiple models," *IEEE Trans. Autom. Control*, vol. 45, no. 9, pp. 1669–1686, Sep. 2000.

[11] A. Karimi and I. D. Landau, "Robust adaptive control of a flexible transmission system using multiple models," *IEEE Trans. Control Syst. Technol.*, vol. 8, no. 2, pp. 321–331, Mar. 2000.

[12] M. Safonov and T.-C. Tsao, "The unfalsified control concept and learning," *IEEE Trans. Autom. Control*, vol. 42, no. 6, pp. 843–847, Jun. 1997.

[13] M. Stefanovic, R. Wang, and M. Safonov, "Stability and convergence in adaptive systems," in *Proc. Amer. Control Conf.*, Boston, MA, 2004, pp. 2012–2021.

[14] A. Dehghani, B. D. O. Anderson, and A. Lanzon, "Unfalsified adaptive control: A new controller implementation and some remarks," in *Proc. Eur. Control Conf.*, Jul. 2007, pp. 709–716.

[15] S. Baldi, G. Battistelli, E. Mosca, and P. Tesi, "Multi-model unfalsified adaptive switching supervisory control," *Automatica*, vol. 46, no. 2, pp. 249–259, Feb. 2010.

[16] A. S. Morse, D. Q. Mayne, and G. C. Goodwin, "Applications of hysteresis switching in parameter adaptive control," *IEEE Trans. Autom. Control*, vol. 37, no. 9, pp. 1343–1354, Sep. 1992.

[17] J. P. Hespanha, D. Liberzon, and A. S. Morse, "Overcoming the limitations of adaptive control by means of logic-based switching," *Syst. Control Lett.*, vol. 49, no. 1, pp. 49–65, Apr. 2003.

[18] S. Fekri, "Robust Adaptive MIMO Control Using Multiple-Model Hypothesis Testing and Mixed μ Synthesis," Ph.D. dissertation, Instituto Superior Tecnico, Lisbon, Portugal, 2005.

[19] S. Fekri, M. Athans, and A. Pascoal, "Issues, progress and new results in robust adaptive control," *Int. J. Adaptive Control Signal Processing*, vol. 20, no. 10, pp. 519–579, Dec. 2006.

[20] S. Fekri, M. Athans, and A. Pascoal, "Robust multiple model adaptive control (RMMAC): A case study," *Int. J. Adaptive Control Signal Processing*, vol. 21, no. 1, pp. 1–30, Feb. 2007.

[21] M. Athans, D. Castanon, K. Dunn, C. Greene, W. Lee, N. Sandell, and A. Willsky, Jr, "The stochastic control of the F-8C aircraft using the multiple model adaptive control (MMAC) method—Part I: Equilibrium flight," *IEEE Trans. Autom. Control*, vol. AC-22, no. 5, pp. 768–780, Oct. 1977.

[22] G. C. Griffin, Jr and P. S. Maybeck, "MMAE/MMAC techniques applied to large space structure bending with multiple uncertain parameters," in *Proc. IEEE 34th Conf. Decision Control*, Dec. 1995, vol. 2, pp. 1153–1158.

[23] G. C. Griffin, Jr and P. S. Maybeck, "MMAE/MMAC control for bending with multiple uncertain parameters," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 33, no. 3, pp. 903–912, Jul. 1997.

[24] G. J. Schiller and P. S. Maybeck, "Control of a large space structure using MMAE/MMAC techniques," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 33, pp. 1122–1131, Oct. 1997.

[25] M. Kuipers and P. Ioannou, "Practical adaptive control: A benchmark example," in *Proc. Amer. Control Conf.*, Seattle, WA, Jun. 2008, pp. 5768–5173.

[26] D. F. Delchamps, "Analytic feedback control and the algebraic Riccati equation," *IEEE Trans. Autom. Control*, vol. 29, no. 11, pp. 1031–1033, Nov. 1984.

[27] R. A. Nichols, R. Reichert, and W. J. Rugh, "Gain scheduling for h-infinity controllers: A flight control example," *IEEE Trans. Control Syst. Technol.*, vol. 1, no. 2, pp. 69–79, Mar. 1993.

[28] R. T. Reichert, "Dynamic scheduling of modern-robust control autopilot designs for missiles," *IEEE Control Syst. Mag.*, vol. 12, no. 5, pp. 35–42, 1992.

[29] M. G. Kellett, "Continuous scheduling of h-infinity controllers for a ms760 paris aircraft," in *Robust Control System Design Using H-Infinity and Related Methods*, P. H. Hammond, Ed. London, U.K.: Institute of Measurement and Control, 1991, pp. 197–223.
[30] R. A. Hyde and K. Glover, "The application of scheduled h-infinity controllers to a vstol airccraft," *IEEE Trans. Autom. Control*, vol. 38, no. 7, pp. 1021–1039, Jul. 1993.
[31] H. Buschek, "Robust autopilot design for future missile systems," in *Proc. AIAA Guidance, Navig., Control Conf.*, 1997, [CD ROM].
[32] J. H. Kelly and J. H. Evers, "An interpolation strategy for scheduling dynamic compensators," in *Proc. AIAA Guidance, Navigation, Control Conf.*, 1997, [CD ROM].
[33] H. Niemann and J. Stoustrup, "An architecture for implementation of multivariable controllers," in *Proc. Amer. Controls Conf.*, San Diego, CA, Jun. 1999, pp. 4029–4033.
[34] D. J. Stilwell and W. J. Rugh, "Stability preserving interpolation methods for the synthesis of gain scheduled controllers," *Automatica*, vol. 36, no. 5, pp. 665–671, 2000.
[35] H. Niemann, J. Stoustrup, and R. B. Abrahamsen, "Switching between multivariable controllers," *Optimal Control Appl. Methods*, vol. 25, pp. 51–66, 2004.
[36] S. M. Shahruz and S. Behtash, "Design of controllers for linear parameter-varying systems by the gain scheduling technique," *J. Math. Anal. Appl.*, vol. 168, no. 1, pp. 195–217, 1992.
[37] T. T. Tay, I. Mareels, and J. B. Moore, *High Performance Control, ser. Systems and Control: Foundations and Applications*. Boston, MA: Birkhuser, 1997.
[38] C. Rohrs, L. Valavani, M. Athans, and G. Stein, "Robustness of continuous-time adaptive control algorithms in the presence of unmodeled dynamics," *IEEE Trans. Autom. Control*, vol. AC-30, no. 9, pp. 881–889, Sep. 1985.
[39] A. S. Morse, "Towards a unified theory of parameter adaptive control: Tunability," *IEEE Trans. Autom. Control*, vol. 35, no. 9, pp. 1002–1012, Sep. 1990.
[40] M. M. Polycarpou and P. A. Ioannou, "On the existence and uniqueness of solutions in adaptive control systems," *IEEE Trans. Autom. Control*, vol. 38, no. 3, pp. 474–479, Mar. 1993.

Matthew Kuipers received the B.S. degree in electrical engineering from Northeastern University, Boston, MA, in 2003 and the M.S. and Ph.D. degrees in electrical engineering from the University of Southern California, Los Angeles, in 2005 and 2009, respectively.

His research interests include robust adaptive control, multiple model approaches, nonlinear control, hypersonic aircraft modeling and control, and intellectual property law.

Petros Ioannou (S'80–M'83–SM'89–F'94) received the B.Sc. degree in mechanical engineering (with first class honors) from the University College London, London, U.K., in 1978 and the M.S. degree in mechanical engineering and Ph.D. degree in electrical engineering from the University of Illinois, Urbana, in 1980 and 1982, respectively.

In 1982, he joined the Department of Electrical Engineering—Systems, University of Southern California, Los Angeles. He is currently a Professor in the same department and the Director of the Center of Advanced Transportation Technologies. He also holds a joint appointment with the Department of Aerospace and Mechanical Engineering. He is the author/coauthor of five books and over 150 research papers in the area of controls, neural networks, nonlinear dynamical systems, and intelligent transportation systems. His research interests are in the areas of adaptive control, neural networks, nonlinear systems, vehicle dynamics and control, intelligent transportation systems, and marine transportation.

Dr. Ioannou received the Outstanding Transactions Paper Award from the IEEE Control Systems Society and the 1985 Presidential Young Investigator Award for his research in Adaptive Control in 1984. He has been an Associate Editor for the IEEE TRANSACTIONS ON AUTOMATIC CONTROL, the *International Journal of Control*, *Automatica*, and the IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS. He also served as a member of the Control System society on IEEE Intelligent Transportation Systems (ITS) Council Committee, and his center on advanced transportation technologies was a founding member of Intelligent Vehicle Highway System (IVHS) America, which was later renamed ITS America. He is currently an Associate Editor at Large of the IEEE TRANSACTIONS ON AUTOMATIC CONTROL and a Chairman of the International Federation of Automatic Control (IFAC) Technical Committee on Transportation Systems.