Reading: Class Notes, Sections 1.6, 6.5, 6.6, 7.1.

Problems:

1. Compute the transfer function of the system
   \[\dot{x}_1 = x_2 - u\]
   \[\dot{x}_2 = -3x_1 - 2x_2 + u\]
   \[y = x_1 + x_2\]

2. Construct minimal realizations of the following transfer functions:
   \[\frac{s - 3}{s^2 - 5s + 6}, \quad \frac{s^2 + 1}{s^3 - 2s^2 + s}\]

3. Consider the system
   \[\dot{x} = -2x + u\]
   \[y = x + u\] (1)
   a) Construct a system of the form
      \[\dot{z} = az + by\]
      \[u = cz + dy\] (2)
      which serves as an inverse to (1), in the sense that if we take an input signal \(u\), feed it into the system (1), compute the output \(y\), and feed this \(y\) into the system (2), we get the original signal \(u\) back as the output of (2) (assuming zero initial conditions for both \(x\) and \(z\)).
      b) Run computer simulations to verify that the inverse you constructed in part a) indeed works as expected. Check what happens if you vary the initial conditions.

4. Consider the system \(\dot{x} = Ax + Bu\), \(y = Cx\) and suppose that it is both controllable and observable. Now consider the feedback of the form \(u = Kx + v\), which leads to the system with new control \(v\):
   \[\dot{x} = (A + BK)x + Bv\]
   \[y = Cx\]
   a) Is the new system controllable? Prove or give a counterexample.
   b) Is the new system observable? Prove or give a counterexample.

5. On Tue Oct 24 in class we will state and partially prove a lemma which says that we can go from a controllable pair \((A, b)\) to its corresponding controllable canonical form \((\bar{A}, \bar{b})\) via a coordinate transformation \(x = Px\). In class we will derive \(P = C(A, b)C^{-1}(A, \bar{b})\) and verify \(\bar{b} = P^{-1}b\), but won’t verify \(\bar{A} = P^{-1}AP\). Finish the proof by verifying this last claim.

6. Prove that any two minimal realizations of a given transfer function \(g(s)\) can be obtained from each other by a coordinate transformation. (Hint: use the result of the previous problem.)

7. Consider the system
   \[\dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 2 & 5 \\ 1 & -1 & -3 \end{pmatrix} x + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} u\]
   Using the algorithm from the Tue Oct 24 class, find a state feedback law \(u = Kx\) such that the poles of the closed-loop system are \(-1\) and \(-2 \pm i\). You can use MATLAB (or any other software) to multiply matrices, invert a matrix, check rank of a matrix, and compute characteristic polynomial, but not for anything else.