ECE 517, Fall 21 **PROBLEM SET 5** **Due Wednesday, Dec 8**

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1. Consider the following adaptive control system.

Plant: $\dot{y} = ay + bu$, where a and $b \neq 0$ are unknown parameters.

Control law: $u = -ky$. Update law for k: $\dot{k} = \hat{b}(\hat{a} - \hat{b}k + 1)$.

(Interpretation: drive k to the equilibrium value $\frac{\hat{a}+1}{\hat{b}}$, but stop if $\hat{b} \to 0$ to keep k bounded.)

Estimator: $\dot{\hat{y}} = -(\hat{y} - y) + \hat{a}y + \hat{b}u$.

Update laws for $\hat{a}$, $\hat{b}$ (as in class): $\dot{\hat{a}} = -\gamma ey$, $\dot{\hat{b}} = -\gamma eu$, where $\gamma > 0$ and $e = \hat{y} - y$.

Show that there always exist initial values of $y, \hat{y}, k, \hat{a}, \hat{b}$ for which we get a trajectory along which $e \equiv 0$ but $y \nearrow \infty$. Interpret this situation in terms of lack of detectability.

2. Simulate the supervisory control scheme for Example 10 as developed in the class notes. You can select a specific uncertainty set $\mathcal{P} \subset \mathbb{R}$ as long as it contains both positive and negative values. Use the multi-estimator and the control laws given in the notes for this example, and the hysteresis switching logic. (You may need to increase the hysteresis constant until you get stable behavior.) Repeat your simulation with an additive disturbance in the plant equation.

3. Consider the system (114) from the class notes. Ignore the z-dynamics and implement the direct MRAC scheme from the notes for the scalar plant (115). Simulate the system that results from applying this direct MRAC controller to the plant (114). Verify the claims given at the end of Section 10 in the notes, namely, that for ε small enough the effect of the z-dynamics is small (but does not in general diminish over time).