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1. Consider the class of scalar plants

$$\dot{y} = ay + bu, \qquad a \in \mathbb{R},\ b > 0 \tag{1}$$

In Section 3.1.1 of class notes, it is shown that the controller (19) is a universal regulator for this class of plants, with the help of the Lyapunov function $V(y) = y^2/2$. For the same controller, find a Lyapunov function more similar to the Lyapunov function (3) used in Example 1 (i.e., one that depends also on a, b, k) such that convergence of y to 0 in closed loop can be proved by a direct application of Theorem 2.

2. Consider again the class of scalar plants (1). Show that there doesn't exist a *linear* universal regulator for this class of plants, i.e., a universal regulator of the form (22) from class notes with f and h linear functions. Here the dimension of z can be arbitrary. (Thus you cannot use the non-existence result for rational controllers proved in class, because it is restricted to scalar z.)

3. Design a universal regulator for the class of scalar plants

$$\dot{y} = a\varphi(y) + bu, \qquad a \in \mathbb{R},\ b > 0$$

where $\varphi(\cdot)$ is a fixed known function. Justify rigorously that it works.

4. Consider a linear system

$$\dot{x} = Ax + Bu$$

and assume that A is Hurwitz, so that we have $\|e^{At}\| \leq ce^{-\lambda_0 t}$ for some $c, \lambda_0 > 0$. Prove the following:

a) If $u \in L_2$ or u is bounded, then x is bounded. (Hint: use the variation-of-constants formula and the Cauchy-Schwartz and Hölder's inequalities.)

b) If $u \in L_2$ or $u \to 0$, then $x \to 0$. (Hint: use part a).)

Note that in both art a) and in part b) you need to consider two separate cases.

5. Simulate the control systems described in Examples 13.16 and 13.17 in Khalil's Nonlinear Systems book (3rd edition, pp. 532–534) and confirm the unstable behavior of closed-loop solutions.