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Robust estimator design for switched systems with unknown switching time: An LMI-based approach

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ABSTRACT

In this paper, a novel method for the design of a robust estimator for a class of switched linear systems subject to unknown inputs is presented. To consider a more general case compared to the literature, the switching sequence is assumed to be minimum average-dwell time but not available for measurement. To deal with this issue, the proposed estimator structure is divided into two blocks: mode-estimator and continuous-estimator. Based on this structure, a bank of robust estimators is designed for each block that is able to simultaneously estimate the active mode of the switched system and states. Using a common Lyapunov function, a sufficient condition in terms of linear matrix inequalities is derived, which guarantees the exponential stability of the estimation error dynamics. Simulation results illustrate the performance of the proposed robust estimator.

1. Introduction

Estimator design for linear systems subject to unknown inputs referred to as Unknown Input Observers (UIO) is one of the interesting topics in system theory that has been studied during the last two decades. Briefly, unknown inputs can contain any modelling errors, plant variations, and disturbances and thus they have a significant impact on the behaviour of the plant and control system. It should be noted that, in case of unknown inputs, typical estimator designs usually fail and consequently the desired control performance cannot be achieved. Regarding UIO and robust estimators in general, we recommend our readers to see [2,9–11,17,24,25] and the references therein, which are some of the recent works in the literature focusing on robust estimator design for both linear and nonlinear systems. But to where the current literature takes us in the field of robust estimator design for switched systems?

The stability and stabilization problems of switched linear systems, as a special class of hybrid systems, have been studied extensively and quite comprehensive results are now available in the literature. See for instance [16,19–22,29]. However, and unlike the stabilization problems via different control techniques, the estimator design problem for switched systems has not been fully investigated. Some of the key references in this area are listed below.

The study of the fundamental properties of switched linear systems has received a great deal of attention during the last decade. In particular, the observability of switched linear systems has been thoroughly analysed depending on whether the switching signal is known or unknown [6]. A detailed characterization of the observability and an observer design method for switched linear systems with state jumps followed by some modifications on relaxing the constraint of the observers have been reported in [27,28]. Undoubtedly, [3] and [4] are two well-known articles in the field of estimator design for hybrid systems where the authors propose a general estimator structure for hybrid systems. The proposed estimator in either reference is composed of a mode-estimator and a continuous-estimator performing together to reconstruct the states under dwell-time constraint on the switching signal. It should be noted that none of the aforementioned works consider unknown inputs in the plant realisation; thus, their design is not robust with respect to unknown inputs. Particularly, for the switched system, the controllability and observability of switched linear systems without unknown inputs are addressed in [26]. In [1], the authors proposed a switched Luenberger estimator and a sufficient condition for stability of the error dynamics. They used a common Lyapunov function and a Linear Matrix Inequality (LMI) approach to obtain a sufficient condition for stability.

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On the other hand from the robust estimator design point of view, there are not so many research studies reported for switched systems in the literature. In [8], a systematic LMI-based procedure for design of a UIO for switched linear discrete-time systems has been proposed. The authors have assumed that the switching signal is available in real time and also there is no jump in the continuous state once a switch takes place. A similar work has been reported in [7], where the authors proposed sufficient conditions in terms of LMIs for the existence of both full-order and reduced-order UIOs. It is worth noting that they also considered a known switching signal and the case that the state does not jump at the switching instants. In [18], robust estimator design for a class of switched nonlinear discrete-time descriptor systems has been studied. However, they also assumed a real-time accessible switching time sequence and no jump in the continuous state. The latter assumption, i.e. having no jump in state, has been removed in [5]. Although they considered a jump in the continuous state and designed an estimator for switched linear systems subject to unknown inputs, they assumed that switching signal is available for real-time measurement.

To the best of authors' knowledge, none of the aforementioned works have addressed the problem of estimator design for switched systems subject to unknown inputs, when the switching signal is also unknown. In this paper, the assumption on the switching signal being real-time measurable is relaxed and our objective is to propose a robust estimator that reconstructs both the active mode of the switched system and the complete continuous states via the knowledge of the switched plant inputs and outputs. Sufficient conditions in terms of LMIs are provided. The feasibility of the LMI conditions guarantees the exponential convergence of the estimation error dynamics in the presence of unknown inputs.

The rest of this paper is organized as follows. In Section 2, we introduce the class of switched systems subject to unknown inputs. In Section 3, a new method for the design of a robust estimator for the class of the switched systems under study is proposed. By using a common Lyapunov function, we propose a sufficient condition in terms of nonlinear matrix inequalities in order to make the error dynamics exponentially stable. The obtained inequalities are then reformulated as a set of LMIs. The effectiveness of the proposed estimator is evaluated through a numerical example. Simulation results are shown in Section 4. Finally, the paper ends with concluding remarks in Section 5.

2. Preliminaries and problem statement

Consider the following class of switched linear systems:

$$Sys.:\begin{cases}\dot{x}(t)=A_{\lambda(t)}x(t)+B_{\lambda(t)}u(t)+E_{\lambda(t)}\nu(t)\\ y(t)=C_{\lambda(t)}x(t)\\ x(t_i^+)=\Phi x(t_i^-);\, i=1,2,\ldots\end{cases}\tag{1}$$

where $x\in R^n$, $u\in R^m$, $\nu\in R^h$ and $y\in R^p$ are the state vector, known inputs, unknown inputs, and measured outputs, respectively. The parameter $\lambda(t)$ is a piecewise-constant discrete state function that takes values on the discrete set $\{1,\ldots,N\}$, where N is the number of "modes" that composes the overall switched dynamics. $q\in\{1,\ldots,N\}$ is the "active mode" at time t if $\lambda(t)=q$, corresponding to the tuple (A_q, B_q, C_q, E_q) with appropriate dimensions. The sequence $\{t_1,t_2,\ldots\}$ denotes the switching times at which the system mode changes (i.e., $\lambda(t_i^+)\neq\lambda(t_i^-)$). The value of the state jump is defined by the matrix Φ, which is assumed to be known at each switching time. To specify the class of linear systems under study, it is assumed that for $q=\{1,2,\ldots,N\}$ the following properties hold.

Assumption 1. All pairs (A_q, C_q) are observable.

Assumption 2. $rank(C_qE_q)=rank(E_q)=h$.

The aim is to design a robust estimator working under an average dwell-time constraint such that it estimates the state vector $x(t)$ in presence of unknown inputs $\nu(t)$.

Definition 1. [15] Let $N_\lambda(t_f, t_0)$ be the number of discontinuities of the switching signal $\lambda(t)$ on the interval (t_0, t_f). We say that $\lambda(t)$ has an average dwell time τ_a if there exists a positive number N_0 such that

$$N_\lambda(t_f,t_0)=N_0+\frac{t_f-t_0}{\tau_a}\quad\forall\quad 0\le t_0\le t_f\tag{2}$$

By definition $N_\lambda(t)=N_\lambda(t,0)$, that is $N_\lambda(t)$ is the number of discontinuities of the switching function $\lambda(t)$ from the initial time instant $t=0$ until the current time t.

3. Estimator design

The structure of the proposed estimator is illustrated in Fig. 1. It is composed of two blocks: a mode-estimator and a continuous-estimator. The mode estimator receives the plant inputs u and outputs y and it provides the estimate $\hat{q}$ of the discrete mode q of the switched plant at the current time. This information is used by the continuous-estimator to construct an estimate $\hat{x}$ of the plant's continuous state that exponentially converges to real state x. Now the question is how to design these estimators? Here the idea is to design a bank of estimators such that each one corresponds to a particular mode. These observers are then used to generate the residuals $r_{\hat{q}}$ for $\hat{q}=\{1,\ldots,N\}$. The computed residuals are then compared with a threshold R_0, which is a design parameter. For a mode estimator, if $r_{\hat{q}}\le R_0$, then $\hat{q}=q$. The continuous estimator corresponding to mode q is chosen to obtain the correct estimation of the state x. Here and similar to the case in field of fault detection, the residual is designed to be either zero or small in a realistic case where the process is subject to noise and model uncertainty, in the UI-free case and deviate significantly from zero when a UI occurs [12,13]. But for the remainder of this paper it is assumed, without loss of generality, that a residual is 0 in the active-mode case. Inspired by this discussion, the following estimator structure is constructed:

$$Est.:\begin{cases}\dot{\xi}_{\hat{q}}(t)=H_{\hat{q}}\xi_{\hat{q}}(t)+G_{\hat{q}}u(t)+L_{\hat{q}}y(t)\\ \hat{x}_{\hat{q}}(t)=\xi_{\hat{q}}(t)-J_{\hat{q}}y(t)\\ r_{\hat{q}}=\|C_{\hat{q}}\hat{x}_{\hat{q}}(t)-y(t)\|\\ \hat{x}(t_i^+)=\Phi\hat{x}(t_i^-);\, i=1,2,\ldots\end{cases}\tag{3}$$

where vector $\xi\in R^n$ and $\hat{x}$ is the estimation of x. The matrices $H_{\hat{q}}$, $G_{\hat{q}}$, $L_{\hat{q}}$, and $J_{\hat{q}}$ for $\hat{q}=\{1,\ldots,N\}$ must be determined such that the error dynamics $e_{q\hat{q}}=x_q-\hat{x}_{\hat{q}}$ exponentially converge to zero. By defining the state estimation error as,

$$\begin{aligned}e_{q\hat{q}}&=x_q-\hat{x}_{\hat{q}}=x_q-\xi_{\hat{q}}+J_{\hat{q}}y\\&=(I+J_{\hat{q}}C_q)x_q-\xi_{\hat{q}}\end{aligned}\tag{4}$$

the error dynamics satisfy,

$$\begin{aligned}\dot{e}_{q\hat{q}}&=H_{\hat{q}}e_{q\hat{q}}+\big(-H_{\hat{q}}(I+J_{\hat{q}}C_q)+(I+J_{\hat{q}}C_q)A_q-L_{\hat{q}}C_q\big)x_q\\&\quad+\big((I+J_{\hat{q}}C_q)B_q-G_{\hat{q}}\big)u+(I+J_{\hat{q}}C_q)E_q\nu\end{aligned}\tag{5}$$

In order to identify the "active mode", each estimator "$\hat{q}$" primarily needs to be designed such that it is sensitive to one real mode "q" and insensitive to others [12–14]. Considering (5) and assuming estimator sensitive to mode "q", i.e. $\hat{q}=q$, and defining $M_{\hat{q}}=I+J_{\hat{q}}C_{\hat{q}}$ if,

$$G_{\hat{q}}=M_{\hat{q}}B_{\hat{q}},\quad M_{\hat{q}}E_{\hat{q}}=0,\quad H_{\hat{q}}M_{\hat{q}}=M_{\hat{q}}A_{\hat{q}}-L_{\hat{q}}C_{\hat{q}}\tag{6}$$

then the error dynamics (5) satisfy,

$$\dot{e}_{q\hat{q}}=\quad H_{\hat{q}}e_{q\hat{q}}\tag{7}$$

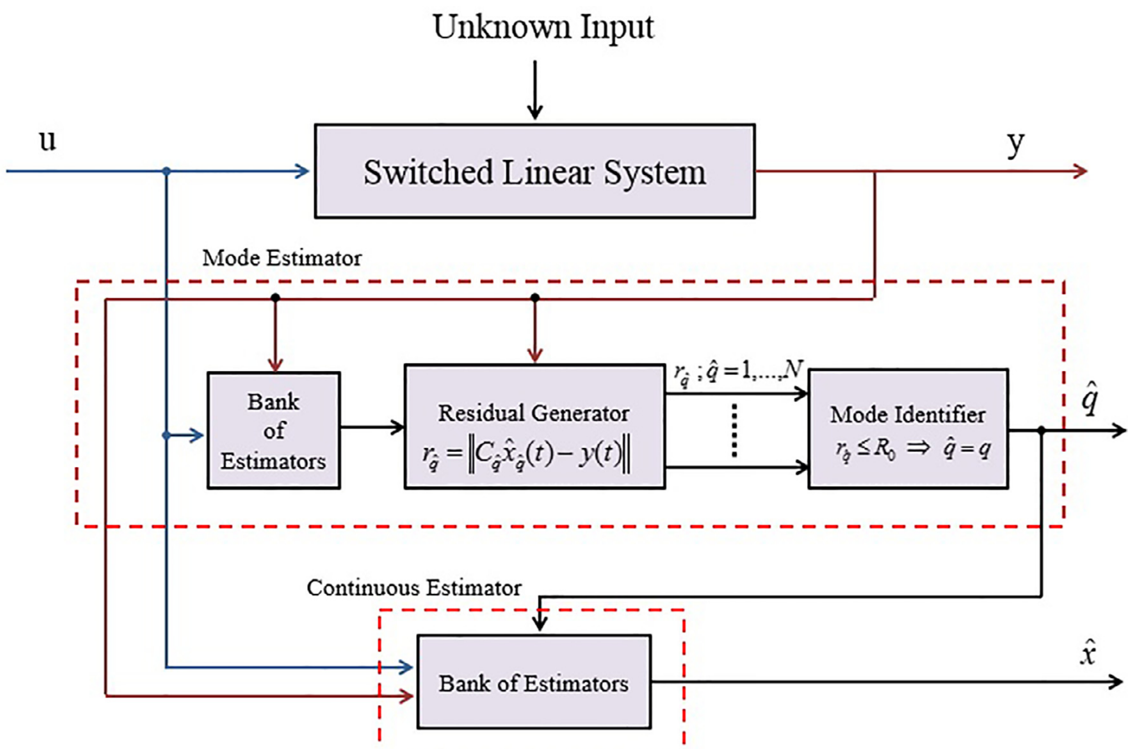

Fig. 1. Overview of estimator structure.

By using the equation $M_{\hat{q}} = I + J_{\hat{q}}C_{\hat{q}}$ and substitution in (6),

$$\begin{aligned} H_{\hat{q}} &= M_{\hat{q}}A_{\hat{q}} - K_{\hat{q}}C_{\hat{q}} \\ L_{\hat{q}} &= K_{\hat{q}}(I + C_{\hat{q}}J_{\hat{q}}) - M_{\hat{q}}A_{\hat{q}}J_{\hat{q}} \end{aligned} \tag{8}$$

where $K_{\hat{q}} = L_{\hat{q}} + H_{\hat{q}}J_{\hat{q}}$. More discussions on how to find the estimator gains will be provided in the sequel. The following theorem gives sufficient conditions for stability of the error dynamics $e_{q\hat{q}}(t)$.

Theorem 1. *Consider the system given in (1) fulfilling Assumptions 1 and 2. For any given scalars $\alpha > 0$ and $\beta > 1$, if there exist matrices $H_1, H_2, \dots, H_N$ and a positive-definite matrix $P > 0$ such that the following inequalities hold*

$$\begin{aligned} &H_{\hat{q}}^T P + PH_{\hat{q}} + 2\alpha P < 0 \\ &\quad \Phi P \Phi - \beta P \le 0 \end{aligned} \tag{9}$$

$\forall \hat{q} \in \{1, \dots, N\}$, then the state estimation error (7) is exponentially stable. Moreover, $\hat{x}(t)$ tends exponentially to $x(t)$ with an α-decaying rate in the presence of the unknown input $v(t)$ provided that the switching sequence fulfils the average dwell-time constraint in (2) with N_0 being an arbitrary positive number and τ_a sufficiently large according to

$$\tau_a > \frac{ln(\beta)}{\alpha} \tag{10}$$

Proof. Consider a Lyapunov function as $V(t) = e^T Pe$ with $P > 0$. By taking the derivative of $V(t)$ along the trajectory of (7),

$$\begin{aligned} \dot{V} &= \dot{e}^T Pe + e^T P\dot{e} \\ &= e^T H_{\hat{q}}^T Pe + e^T PH_{\hat{q}}e \\ &= e^T \left(H_{\hat{q}}^T P + PH_{\hat{q}}\right)e \end{aligned} \tag{11}$$

Therefore, to achieve the globally exponential stability of the error dynamics, the following inequality guaranties $\dot{V} < -2\alpha V$:

$$H_{\hat{q}}^T P + PH_{\hat{q}} + 2\alpha P < 0 \tag{12}$$

where α is the decay rate of V. It should be noted that in each interval $[t_i, t_{i+1})$ the time derivative of V along the trajectories of the switched system (1) fulfils the following chain of inequalities:

$$\dot{V}(t) \le -2\alpha V(t) \quad \Rightarrow \quad V(t) \le e^{-\alpha(t_{i+1}-t_i)} V(t_i^+) \tag{13}$$

On the other hand, we assume that there exists a positive constant $\beta > 1$ such that:

$$V(t_i^+) \le \beta V(t_i^-) \tag{14}$$

which is equivalent to

$$\begin{aligned} &V(t_i^+) - \beta V(t_i^-) \le 0 \\ &e^T(t_i^+)Pe(t_i^+) - \beta e^T(t_i^-)Pe^T(t_i^-) \le 0 \\ &e^T(t_i^-)\Phi P\Phi e(t_i^-) - \beta e^T(t_i^-)Pe^T(t_i^-) \le 0 \\ &e^T(t_i^-)(\Phi P\Phi - \beta P)e^T(t_i^-) \le 0 \end{aligned} \tag{15}$$

which would be implied by

$$\Phi P\Phi - \beta P \le 0 \tag{16}$$

Therefore, the feasibility of (16) implies that $V(t_i^+) \le \beta V(t_i^-)$ holds. Combining (13) and (16), we conclude that

$$V(t_{i+1}^+) \le \beta e^{(-\alpha(t_{i+1}-t_i))} V(t_i^+) \tag{17}$$

By iterating (17) from $i = 0$ to $i = N_\lambda(t) - 1$, the following inequality is obtained:

$$V(t^-) \le V(t_{N_\lambda(t)}) \le \beta^{N_\lambda(t)} e^{-\alpha t} V(0) \tag{18}$$

or equivalently,

$$V(t^-) \le \beta^{N_0} e^{(\frac{ln\beta}{\tau_a} - \alpha)t} V(0) \tag{19}$$

Therefore, we conclude that if τ_a satisfies the bound (10), then (19) exponentially converges to zero as t tends to infinity, which in turns implies that $e(t)$ exponentially converges to zero. □

To design the estimator, it is necessary to find matrices $J_{\hat{q}}$, $K_{\hat{q}}$, and $P > 0$ such that the inequalities given in (9) are satisfied. To

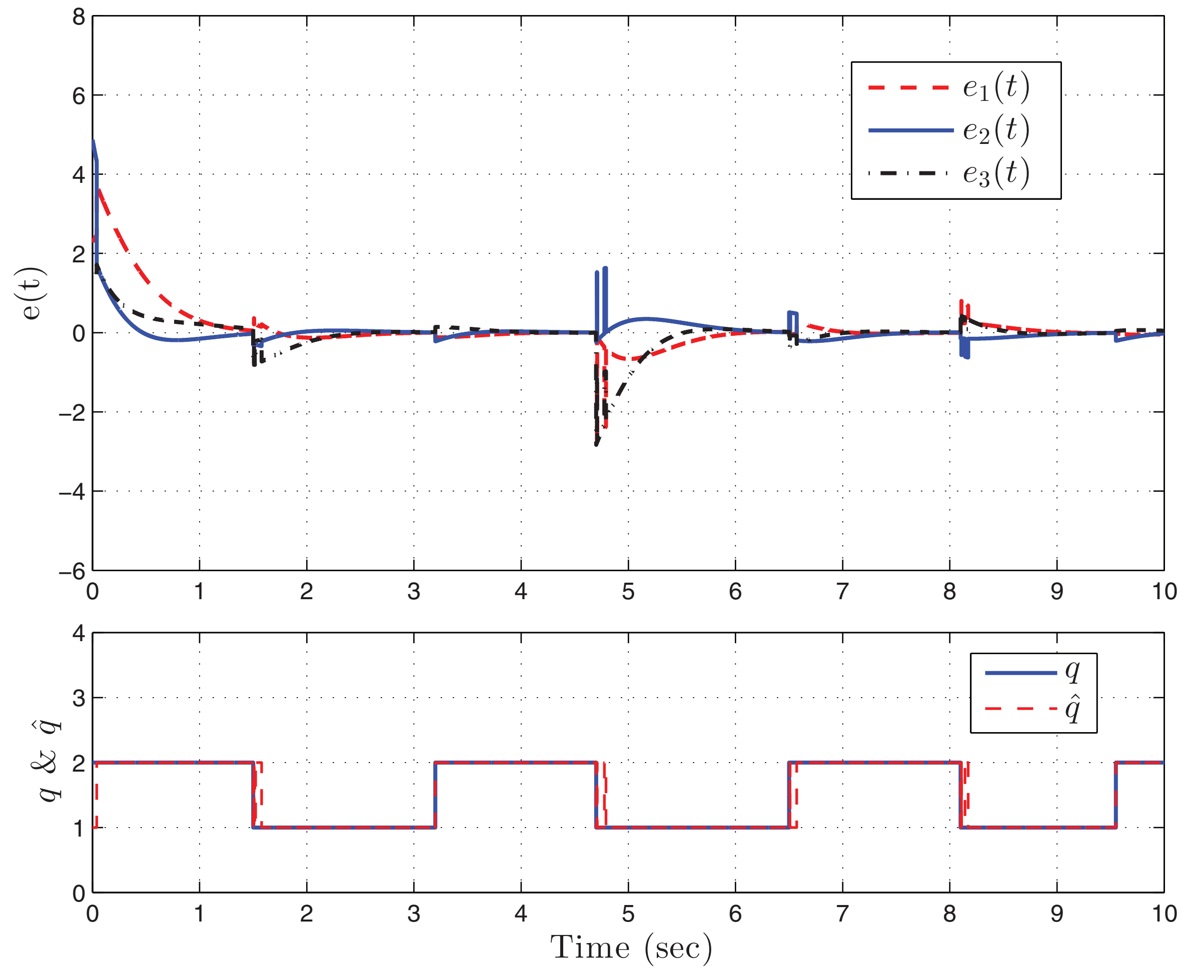

Fig. 2. Estimation errors $e(t)$, switching signal $\lambda(t)$, and its estimate.

tackle this problem, (9) is converted to a standard LMI problem and get a feasible solution through. From (6), $M_{\hat{q}}E_{\hat{q}} = 0$ leads to

$$J_{\hat{q}}C_{\hat{q}}E_{\hat{q}} = -E_{\hat{q}} \tag{20}$$

a general solution of $J_{\hat{q}}$ is as follows,

$$J_{\hat{q}} = -E_{\hat{q}}(C_{\hat{q}}E_{\hat{q}})^{+} + Y_{\hat{q}}(I - (C_{\hat{q}}E_{\hat{q}})(C_{\hat{q}}E_{\hat{q}})^{+}) \tag{21}$$

where $Y_{\hat{q}}$ is an arbitrary matrix of appropriate dimensions and $[.]^{+}$ represents generalized inverse operator satisfying $[.][.]^{+}[.] = [.]$. For notational convenience, (21) is rewritten as

$$J_{\hat{q}} = U_{\hat{q}} + Y_{\hat{q}}V_{\hat{q}} \tag{22}$$

where

$$U_{\hat{q}} = -E_{\hat{q}}(C_{\hat{q}}E_{\hat{q}})^{+}, \quad V_{\hat{q}} = I - (C_{\hat{q}}E_{\hat{q}})(C_{\hat{q}}E_{\hat{q}})^{+} \tag{23}$$

The following theorem is a consequence of the above discussion.

Theorem 2. *Consider system (1) fulfilling Assumptions 1 and 2. For any given scalars $\alpha > 0$ and $\beta > 1$, if there exist real matrices $\bar{K}_{\hat{q}}$, $\bar{Y}_{\hat{q}}$, and P with appropriate dimensions such that the following LMIs hold*

$$\begin{aligned} &PA_{\hat{q}} + A_{\hat{q}}^{T}P + PU_{\hat{q}}A_{\hat{q}} + A_{\hat{q}}^{T}U_{\hat{q}}^{T}P + \bar{Y}_{\hat{q}}V_{\hat{q}}A_{\hat{q}} + A_{\hat{q}}^{T}V_{\hat{q}}^{T}\bar{Y}_{\hat{q}}^{T} \\ &\qquad - P\bar{K}_{\hat{q}}C_{\hat{q}} - C_{\hat{q}}^{T}\bar{K}_{\hat{q}}^{T} + 2\alpha P < 0, \\ &\qquad \Phi P \Phi - \beta P \leq 0 \end{aligned} \tag{24}$$

$\forall \hat{q} \in \{1, \dots, N\}$ where $K_{\hat{q}} = P^{-1}\bar{K}_{\hat{q}}$ and $Y_{\hat{q}} = P^{-1}\bar{Y}_{\hat{q}}$, then the state estimation error (7) is exponentially stable for the correct mode and $\hat{x}(t)$ tends exponentially to $x(t)$ with an α-decaying rate in the presence of unknown input $\nu(t)$ provided that the switching sequence fulfils the average dwell-time constraint in (2) with N_0 being an arbitrary positive number and τ_a sufficiently large according to

$$\tau_a > \frac{\ln(\beta)}{\alpha} \tag{25}$$

Furthermore, the observer gains in (3) are obtained from

$$\begin{aligned} J_{\hat{q}} &= U_{\hat{q}} + Y_{\hat{q}}V_{\hat{q}} \\ M_{\hat{q}} &= I + J_{\hat{q}}C_{\hat{q}} \\ H_{\hat{q}} &= M_{\hat{q}}A_{\hat{q}} - K_{\hat{q}}C_{\hat{q}} \\ L_{\hat{q}} &= K_{\hat{q}}(I + C_{\hat{q}}J_{\hat{q}}) - M_{\hat{q}}A_{\hat{q}}J_{\hat{q}} \end{aligned} \tag{26}$$

Proof. By substituting (22) in (9), we obtain

$$\begin{aligned} H_{\hat{q}}^{T}P + PH_{\hat{q}} + 2\alpha P &= (A_{\hat{q}} + U_{\hat{q}}A_{\hat{q}} + Y_{\hat{q}}V_{\hat{q}}A_{\hat{q}} - K_{\hat{q}}C_{\hat{q}})^{T} \\ &\quad + P(A_{\hat{q}} + U_{\hat{q}}A_{\hat{q}} + Y_{\hat{q}}V_{\hat{q}}A_{\hat{q}} - K_{\hat{q}}C_{\hat{q}}) + 2\alpha P \\ &= A_{\hat{q}}^{T}P + A_{\hat{q}}^{T}U_{\hat{q}}^{T}P + A_{\hat{q}}^{T}V_{\hat{q}}^{T}Y_{\hat{q}}^{T}P - C_{\hat{q}}^{T}K_{\hat{q}}^{T}P \\ &\quad + PA_{\hat{q}} + PU_{\hat{q}}A_{\hat{q}} + PY_{\hat{q}}V_{\hat{q}}A_{\hat{q}} - PK_{\hat{q}}C_{\hat{q}} + 2\alpha P \end{aligned} \tag{27}$$

Since the matrices $K_{\hat{q}}$, $Y_{\hat{q}}$, and P in the inequality above are unknown, it is not yet an LMI with respect to $K_{\hat{q}}$, $Y_{\hat{q}}$, and P. Schur complement and the new variable definitions $K_{\hat{q}} = P^{-1}\bar{K}_{\hat{q}}$ and $Y_{\hat{q}} = P^{-1}\bar{Y}_{\hat{q}}$ help us to sort this out; thus, (24) is obtained. □

Remark 3.1. In order to improve the performance of the designed estimator, it is suggested to have two parallel and similar estimators but with different α-decaying rate; one fast (mode-estimator)

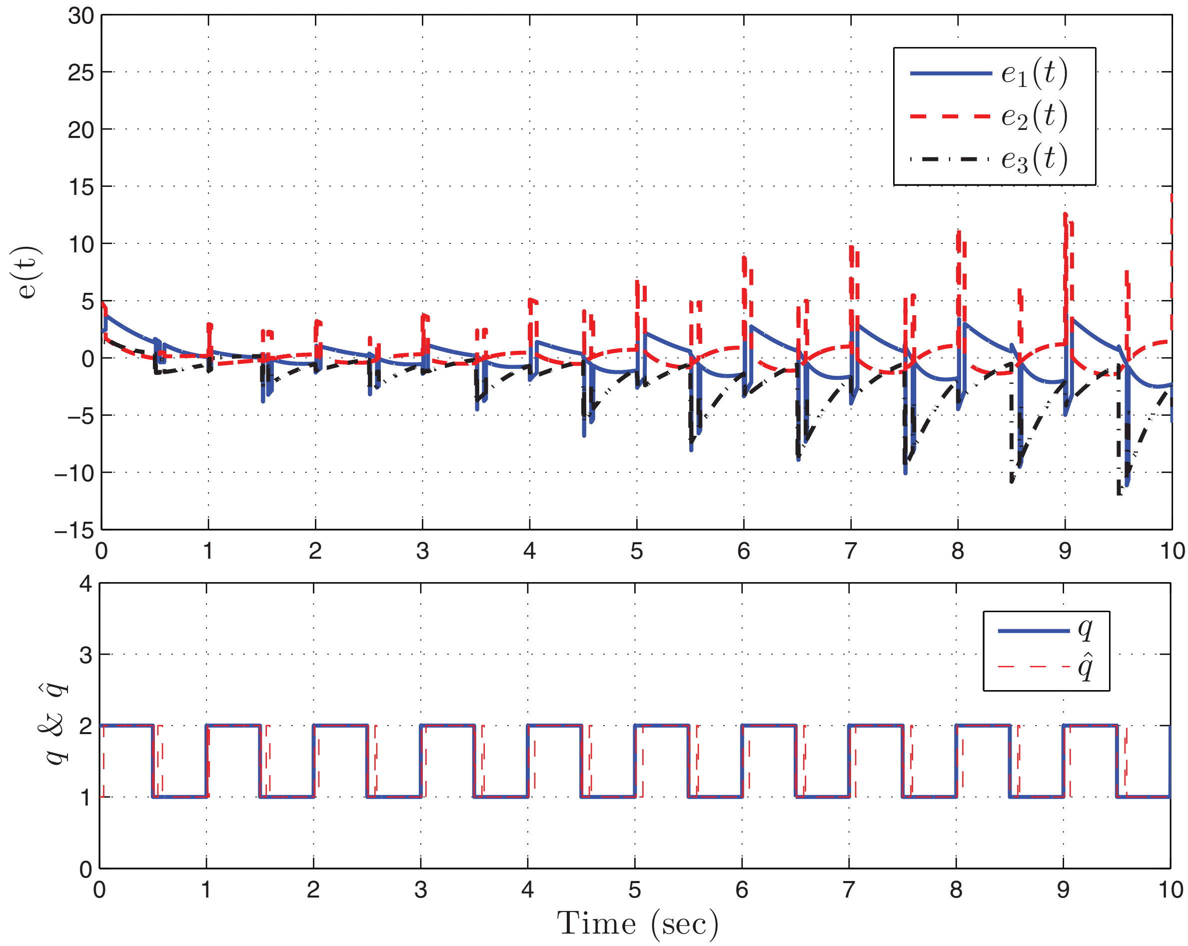

Fig. 3. Estimation errors $e(t)$, switching signal $\lambda(t)$, and its estimate.

and the other one smooth (continuous-estimator). High value of α-decaying rate in mode-estimator makes it capable to identify the correct active mode quickly. As a result, the designed estimator is able to deal with switched systems with small average-dwell time. However, reconstructing the continuous states using this estimator lead to a peaking phenomenon. To tackle this issue, a similar structure with a smaller value of α is used to reconstruct the continuous states using the identified mode obtained from the mode-estimator. In the next section, the peaking phenomenon is shown to emphasize why a smooth continuous-estimator is required. In summary, the estimator design algorithm is itemized as follows:

- Check Assumptions 1 and 2.
- Compute U and V by (23) for $\hat{q} = 1, \ldots\ldots, N$.
- Solve a set of LMIs defined in (24) for $\bar{Y}_{\hat{q}}$, $\bar{K}q$, and P.
- Compute $Y_{\hat{q}} = P^{-1}\bar{Y}_{\hat{q}}$ and $K_{\hat{q}} = P^{-1}\bar{K}_{\hat{q}}$.
- Using $Y_{\hat{q}}$ and $K_{\hat{q}}$, compute the observer gains as (26).

4. Illustrative example

In this section, the proposed robust estimator design is illustrated via a simulation example. Consider the following two-mode switched system:

$$\text{1st Mode}: \begin{cases} \dot{x}(t) = \begin{bmatrix} -1 & 2 & 2 \\ 0 & -2 & 1 \\ -1 & 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} v(t) \\ y(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) \end{cases}$$

$$\text{2nd Mode}: \begin{cases} \dot{x}(t) = \begin{bmatrix} -2 & 1 & 0 \\ -3 & -1 & 1 \\ 1 & -2 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} v(t) \\ y(t) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} x(t) \end{cases}$$

where,

$$\Phi = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{28}$$

It can be easily shown that both Assumptions 1 and 2 are satisfied. The provided LMIs in Theorem 2 have been solved using the YALMIP toolbox [23] (SDPT3 solver) resulting in a feasible solution; therefore, the following estimators are constructed:

- *Mode-estimator 1: ($\alpha = 1000, \beta = 5$)*

$$\begin{cases} \dot{\xi}_{m,1}(t) = \begin{bmatrix} -538.1 & 0.57 & 45.9 \\ 2.4 & -540.6 & -88.9 \\ -50.1 & 98.5 & -535.2 \end{bmatrix} \xi_{m,1}(t) \\ \qquad + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 24540 & 50 \\ -47720 & -90 \\ -136610 & -530 \end{bmatrix} y(t) \\ \hat{x}_{m,1}(t) = \xi_{m,1}(t) - \begin{bmatrix} 21.9 & 0 \\ -44.5 & -1 \\ -266.1 & 0 \end{bmatrix} y(t) \\ r_{m,1} = \|C_1 \hat{x}_{m,1}(t) - y(t)\| \end{cases}$$

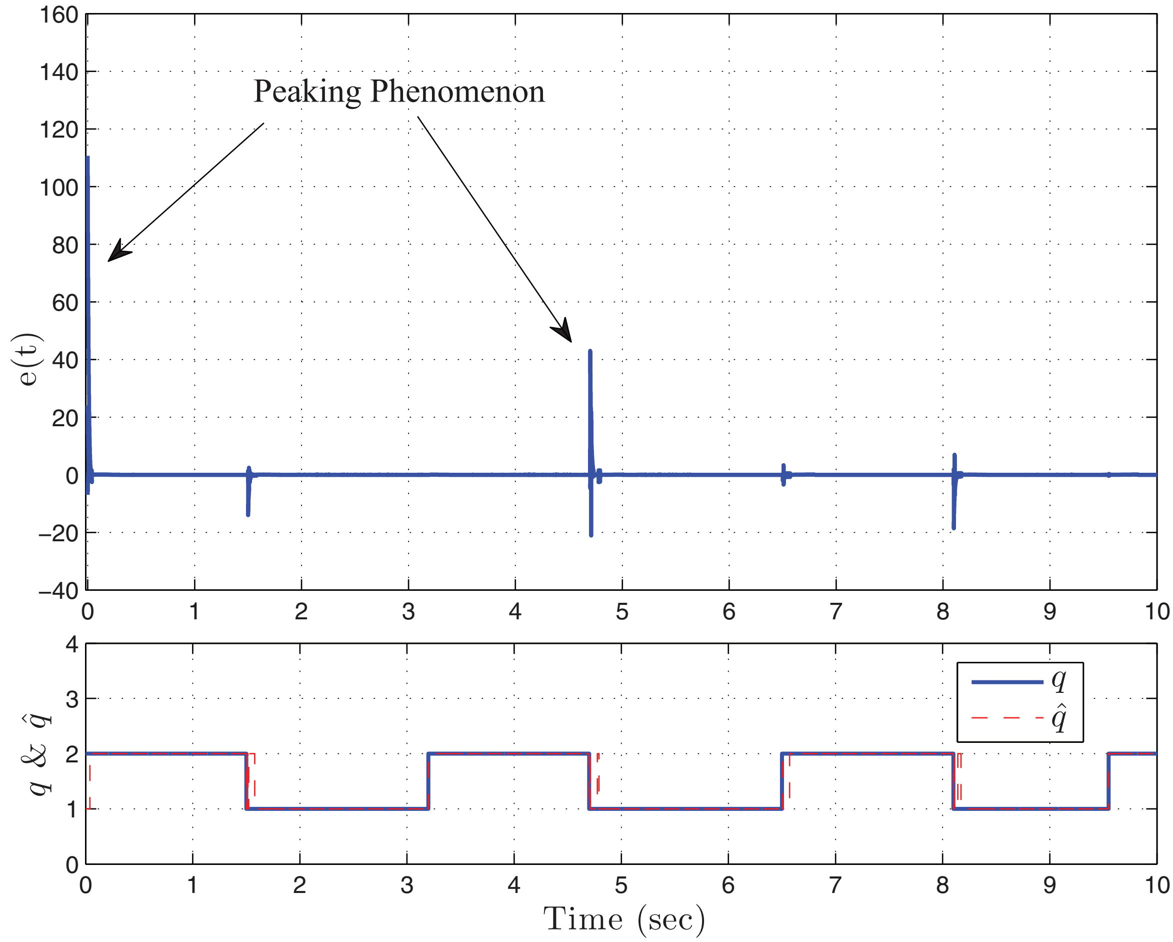

Fig. 4. Peaking phenomenon due to high gain mode-estimator.

- *Mode-estimator 2: ($\alpha = 1000, \beta = 5$)*

$$\begin{cases} \dot{\xi}_{m,2}(t) = \begin{bmatrix} -535.5 & -22.9 & -21.7 \\ 23.5 & -537.8 & 15.3 \\ 21.2 & -13.9 & -537.6 \end{bmatrix} \xi_{m,2}(t) \\ \qquad + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 33783 & -34063 \\ -42113 & 42424 \\ -46316 & 46642 \end{bmatrix} y(t) \\ \hat{x}_2(t) = \xi_{m,2}(t) - \begin{bmatrix} 69.6 & -70.6 \\ -78.4 & 78.5 \\ -81.3 & 81.3 \end{bmatrix} y(t) \\ r_{m,2} = \|C_2 \hat{x}_{m,2}(t) - y(t)\| \end{cases}$$

- *Continuous-estimator 1: ($\alpha = 2, \beta = 5$)*

$$\begin{cases} \dot{\xi}_{c,1}(t) = \begin{bmatrix} -2.5 & 0.6 & 1.7 \\ -0.6 & -2.5 & -0.9 \\ -1.7 & 0.9 & -2.5 \end{bmatrix} \xi_{c,1}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) \\ \qquad + \begin{bmatrix} 1.1 & 1.7 \\ 0.1 & -0.9 \\ 1.2 & 0.5 \end{bmatrix} y(t) \\ \hat{x}_{c,1}(t) = \xi_{c,1}(t) - \begin{bmatrix} -0.1 & 0 \\ -0.4 & -1 \\ 0.3 & 0 \end{bmatrix} y(t) \end{cases}$$

- *Continuous-estimator 2: ($\alpha = 1000, \beta = 5$)*

$$\begin{cases} \dot{\xi}_{c,2}(t) = \begin{bmatrix} -2.5 & 0.6 & 1.7 \\ -0.6 & -2.5 & -0.9 \\ -1.7 & 0.9 & -2.5 \end{bmatrix} \xi_{c,2}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) \\ \qquad + \begin{bmatrix} 1.1 & 1.7 \\ 0.1 & -0.9 \\ 1.2 & 0.5 \end{bmatrix} y(t) \\ \hat{x}_{c,2}(t) = \xi_{c,2}(t) - \begin{bmatrix} -0.1 & 0 \\ -0.4 & -1 \\ 0.3 & 0 \end{bmatrix} y(t) \end{cases}$$

In order to simulate the designed robust estimator, initial values of $x(0) = [1, 1, 1]^T$ and $z(0) = [3, 3, 3]^T$ are assumed. For $u(t)$ being a step signal with amplitude 0.5 and $\nu(t) = \sin(t)$, simulation results are depicted in Figs. 2–4. Fig. 2 shows the real and estimated switching sequence together with the states for the given initial conditions. Based on the figures, one can see that the estimator performs as expected.

Analysis 1: For a different switching sequence which does not fulfil the average dwell time constraint $\tau_a > 0.8$, the estimator does not perform well as shown in Fig. 3 that clearly shows the divergence of the error dynamics. It is worth to note that even in this case, the mode-estimator shows a satisfactory result.

Analysis 2: As was discussed in Remark 3.1, reconstructing the continuous states using a single high speed estimator results in peaking phenomenon. Fig. 4 shows the peaking phenomenon due to high gain of the designed estimator.

5. Conclusions and future works

We considered a novel method to design a robust estimator for a class of switched linear systems subject to unknown inputs. In the class of systems under study, the switching time is unknown and the proposed estimator is able to identify the active mode of the switched system together with the state variables, simultaneously. By assuming an average dwell-time for every switching sequence and using a common Lyapunov function, a sufficient condition is derived which guarantees exponential stability for the error dynamics. This sufficient condition is based on the feasibility of a certain set of LMIs. Simulation results illustrate the performance of the proposed estimator. An interesting direction for future research is extending the method to a reduced-order observer design. Moreover, relaxing Assumption 2, which is a typical constraint in UIO design, would be another potential future path to be considered.

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