A Hybrid System using Two Cascaded Regulators (Tap Changers)

Vaibhav Donde
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Abstract: This term paper investigates behavior of a hybrid system composed of two cascaded "load tap changer transformers". One of these devices changes taps continuously and the other, discretely. The continuous and discrete dynamics interact to produce interesting behavior in parameter space. The results are compared with the paper [1], which looks into a similar problem without discrete dynamics.

1. Introduction

Power systems are nonlinear dynamical systems that are described by coupled differential and algebraic equations. Devices like tap changers and protective relays introduce discrete behavior in the system. Tap changers regulate voltage at two different levels and is achieved by varying the tap ratio 'r_1' (fig 1). X_1 is a reactance of the tap changer. Protective relays change the topology of the system by switching a transmission line in or out of the network. In either case, system description changes at a switching event (tap changing or protective relay switching) and it moves into another region of operation. All such devices operate to provide stability and quality operation. However, the switching events interposed between continuous dynamics may lead to undesirable limit cycles or even instability.

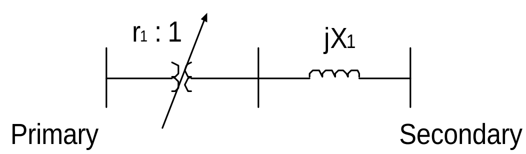


Fig 1: a tap changer

2. Problem description

Fig 2 shows two cascaded tap changers. The source is an infinite bus with a constant complex voltage V_G. The load is represented as a constant admittance G. The system is drawn schematically in fig 3.

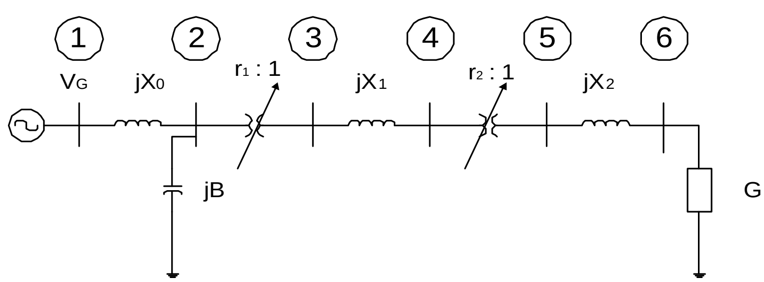


Fig 2: problem network

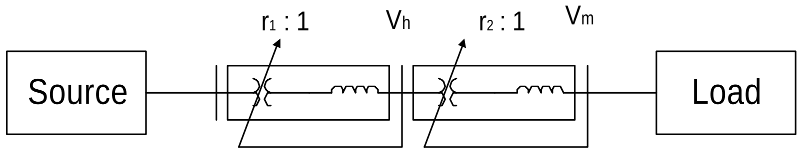


Fig 3: schematic of fig 2

The simplest case is to assume that both the tap changers vary the tap ratios (r_1 and r_2) continuously as defined by the differential equation below.

$$\dot{r}_1 = \frac{1}{T_1}(V_h - V_h^{\,0})$$

$$\dot{r}_2 = \frac{1}{T_2}(V_m - V_m^{\,0})$$

Where T_1, T_2, V_h^0 and V_m^0 are the known constants. This system is simulated and analyzed in parameter and state space in [1]. The authors of [1] observe monotonic behavior, homoclinic loop bifurcation, emergence of unstable limit cycle and Hopf bifurcation for different values of the parameters T_1 and T_2. The problem is reformulated in this paper considering that tap changer 2 varies taps discretely rather than continuously, with a conjecture of observing similar results and phase portraits.

3. Hybrid system and Differential-Algebraic-Discrete (DAD) System Model

Systems that involve both continuous and discrete event dynamics are called hybrid systems. They are characterized by continuous and discrete states; continuous dynamics, discrete events or triggers and a mapping that defines the evolution of discrete states at events [2]. A hybrid power system can be represented by the following set of equations. Note that a power system, in particular, is characterized by differential-algebraic-equations (DAE).

$$\dot{x} = f(x, y, z, p)$$

$$0 = g(x, y, z, p)$$

$$0 = \begin{cases} g^{(i-)}(x, y, z, p) & y_i < 0 \\ g^{(i+)}(x, y, z, p) & y_i > 0 \end{cases}$$

$$z^{+} = h(x^{-}, y^{-}, z^{-}, p) \qquad y_j = 0$$

$$\dot{z} = 0$$

Where *x, y, z, p* are continuous, algebraic, discrete variables and parameters respectively. y_i and y_j are the sets of algebraic variables which define switching events. *h* is a function that maps *z* variables before and after an event. Refer [2] for details.

4. Mathematical description of the problem

The system has 3 prominent components viz. continuous tap changer, discrete tap changer and the remaining power system. These are described individually as follows.

Continuous tap changer (tap changer 1)

- Differential equations

$$\dot{x}_1 = \frac{1}{T_1}(V_h - V_h^{\,0})$$

x_1 : tap ratio of tap changer 1 i.e. r_1

T_1 : time constant of tap changer 1

Discrete tap changer (tap changer 2)

- Differential equations

$$\dot{x}_2 = 1$$

- Algebraic equations

$$y_1 = x_2 - (z_1 + T_2)$$

$$y_2 = V_m - V_m^{\,0}$$

$$y_3 = -1 \qquad ; y_2 < 0$$

$$y_3 = +1 \qquad ; y_2 > 0$$

- Discrete equations

$$z_2^{+} = z_2^{-} + y_3 \times z_{step} \qquad ; y_2 = 0$$

Reset timer

x_2 : timer state

z_1 : time when timer is reset by a tap change

T_2 : tap delay of tap changer 2

y_2 : event variable that decides if taps should be incremented or decremented

z_2 : tap ratio of tap changer 2

y_3 : increases or decreases taps

Power system

- Algebraic equations
  Voltage equality constraints (includes V_h, V_m variables)
  Current equality constraints

The discrete tap changer logic (increment/decrement taps) that defines its DAD equations is as shown in fig 4. It is evident from the flowchart in fig 4 that the switching logic will make the system eventually oscillatory and we can hope to observe stable limit cycles instead of stable equilibrium points, if the trajectories converge.

5. Simulation

The system using the switching logic in fig 4 is simulated for various combinations of the parameters T_1 and T_2. Data as in [1] is used for simulations. Phase portraits and the trajectories of the load voltage (algebraic variable) are drawn. Phase plots are compared with those in [1] with a continuous tap changer. The similarity and differences are noted.

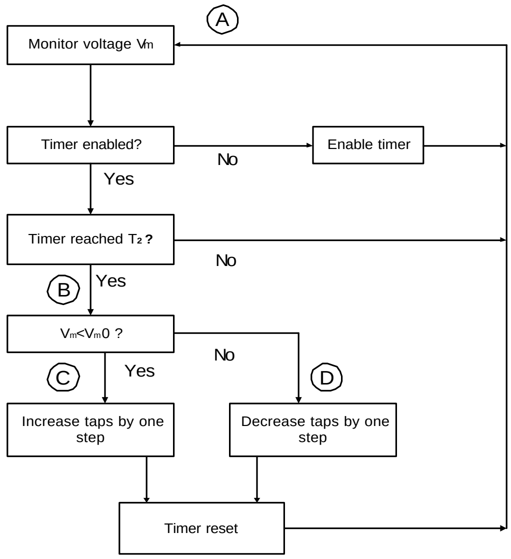
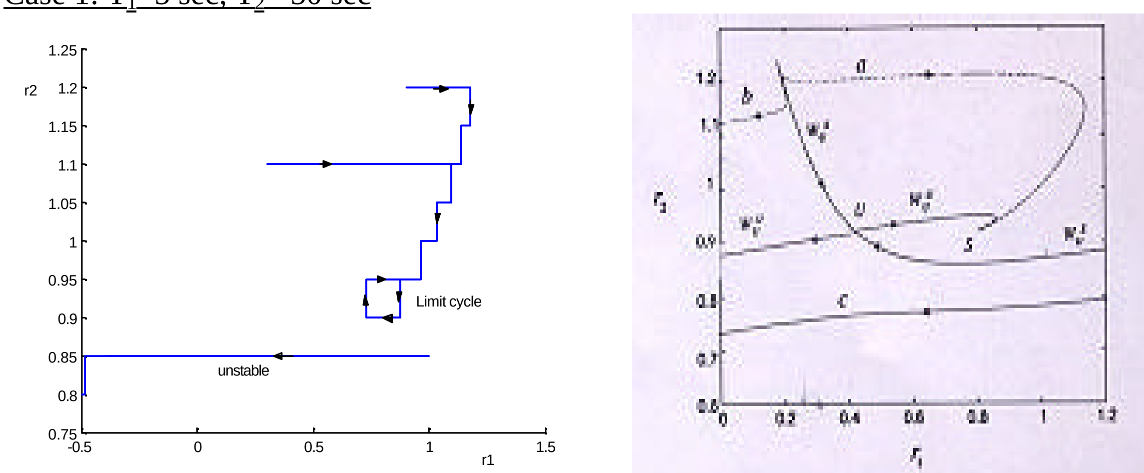


Fig 4: tap changing logic

Case 1: T_1=5 sec, T_2= 50 sec



(a) (b)

Fig 5: (a) phase portrait of the hybrid system

(b) phase portrait of corresponding continuous system in [1]

Fig 5 (a) shows a phase portrait of the hybrid system. It is compared with that with a continuous tap changer system (taken from [1]). A limit cycle is observed in fig 5 (a) as expected. It is a kind of chattering in the system, as there is no provision to attain an equilibrium point. Nevertheless, the position of the limit cycle matches closely with the equilibrium point in fig 5 (b). The trajectories are the horizontal and vertical lines since r_2 is a discrete variable. r_1 moves horizontally as defined by the value of r_2.

Fig 6 shows load voltage trajectories for different initial points, corresponding to fig 5 (a). A stable limit cycle is observed.

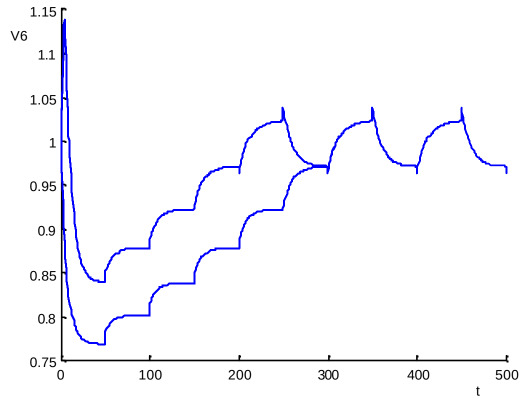
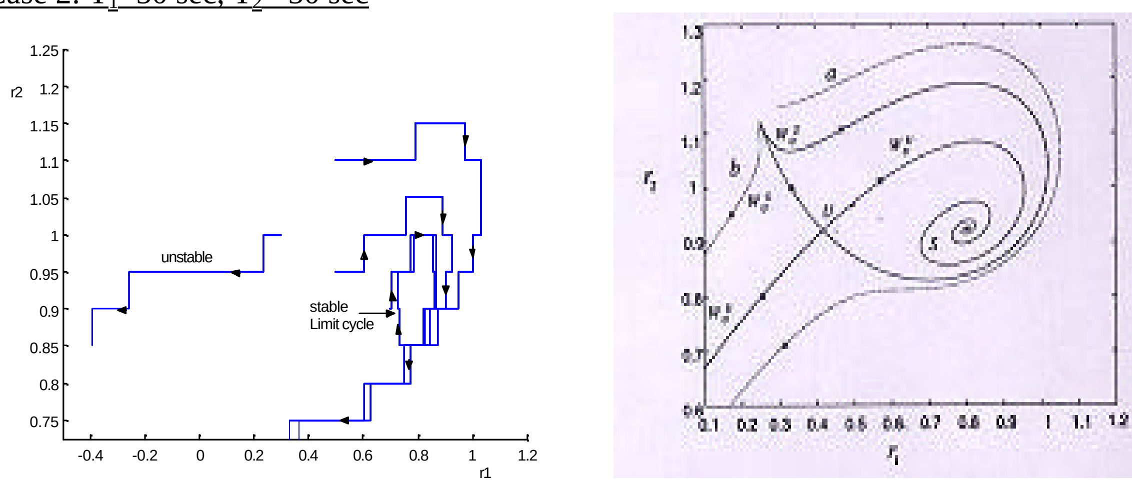


Fig 6: load voltage trajectories

Case 2: T_1=50 sec, T_2= 50 sec



(a) (b)

Fig 7: (a) phase portrait of the hybrid system
(b) phase portrait of corresponding continuous system in [1]

For this set of parameters T_1 and T_2, a stable limit cycle is observed as in case 1. The period of the limit cycle is greater than in case 1. Fig 8 shows load voltage trajectories.

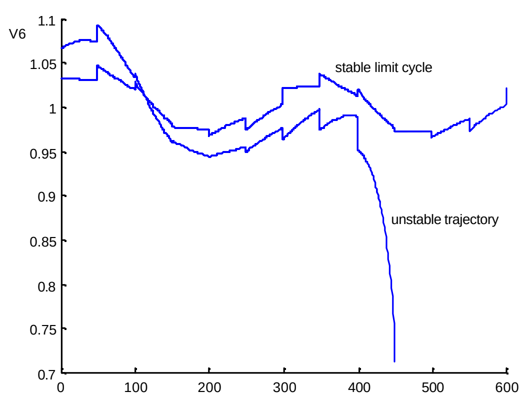
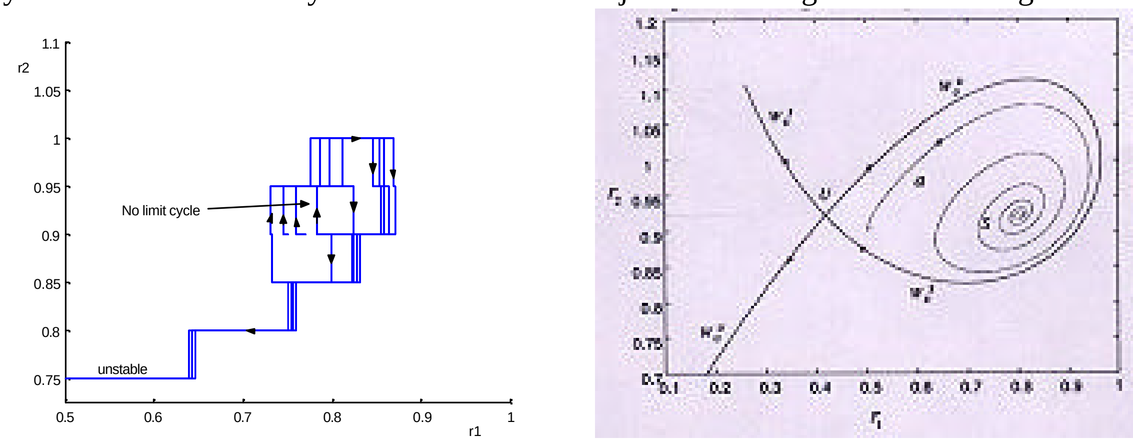


Fig 8: load voltage trajectories

Case 3: T_1=64.4, T_2= 50 sec

For this combination of parameter values, homoclinic loop bifurcation occurs in the case of continuous system considered in [1]. It is not clear whether it happens in our hybrid system. No stable limit cycle is observed. All trajectories diverge as shown in fig 9.



(a) (b)

Fig 9: (a) phase portrait of the hybrid system

(b) phase portrait of corresponding continuous system in [1]

Limit cycles are generally undesirable in power systems. Voltages at different locations in the system are desired to reach nice equilibrium states (near 1 per unit) quickly. A deadband is now introduced in the switching logic that avoids the operation of the tap changer if the system is near a stable equilibrium point. Please refer fig 4 to read fig 10.

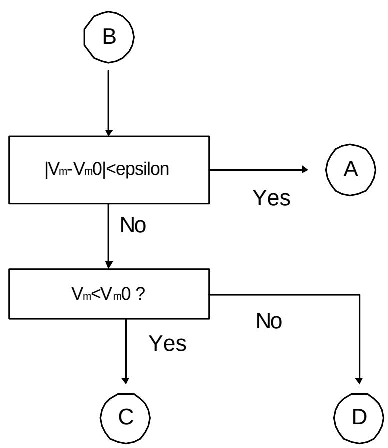
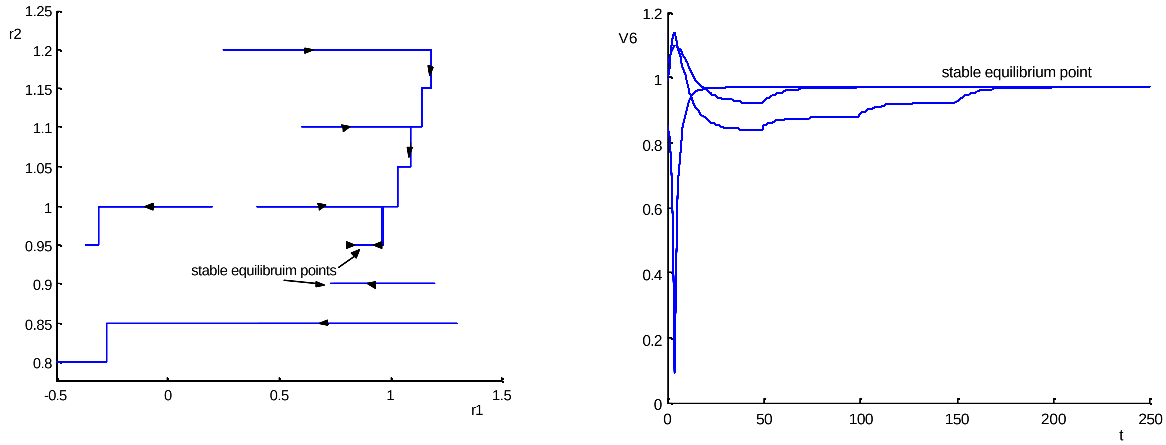


Fig 10: introduction of a deadband in tap changing logic

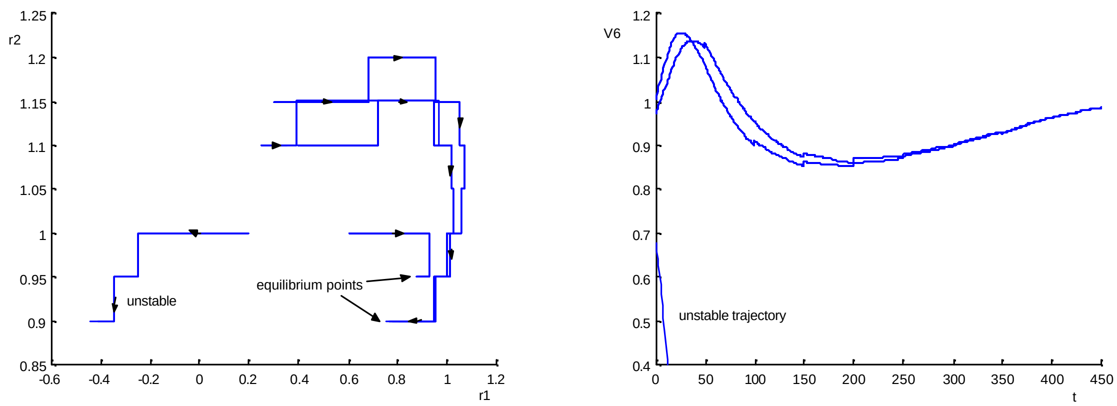
The value of epsilon defines the closeness of the system to equilibrium. Simulation is now run implementing the deadband (fig 10). The system reaches stable equilibrium points instead of limit cycles as expected. Two equilibrium points are found due to the discrete behavior of r_2 as in fig 11 (T_1=5 sec, T_2= 50 sec). Fig 11 (a) may be compared with fig 5. A stable equilibrium point is evident from the stable load voltage trajectory in fig 11 (b).



(a) (b)

Fig 11: (a) phase portrait of the hybrid system
(b) load voltage trajectories

Fig 12 is drawn for T_1=50 sec, T_2= 50 sec (comparable with case 2). Comparing it with fig 7 (b), we see that equilibrium points match closely. Fig 12 (b) shows stable voltage trajectories, with unstable trajectory at bottom left.



(a) (b)

Fig 12: (a) phase portrait of the hybrid system
(b) load voltage trajectories

6. Multiple lyapunov functions: ideas and applications

The nonlinear non-smooth dynamics of hybrid systems make analysis difficult. The stability of hybrid systems may be examined using multiple lyapunov functions. Different lyapunov functions may be chosen for analysis in different regions of system operation defined by switching events. Care must be taken while choosing the lyapunov functions for a given hybrid system so that the combined function decreases along a switching boundary i.e., the energy of the system decreases along the switching boundary.

7. Summary and conclusions

Hybrid systems have limit cycles and other structural properties, as do the continuous dynamic systems. The limit cycles and equilibrium points may be stable or unstable according to the switching details and stability properties of individual systems. New limit cycles may be introduced because of the switching events. One interesting observation in hybrid systems is that the hybrid trajectories may intersect each other as the description of the system changes after a switching event, which does not happen in case of a continuous dynamical system.

8. Unresolved issues/ future work

1. Construction of multiple lyapunov function for a DAD system is not transparent. [3-6] contain related work. Analytical study of a hybrid system in parameter space to observe bifurcations and limit cycles may be attempted.
2. Detection and study of unstable limit cycles of hybrid systems may not be possible with simulations. Poincaré map technique may be exploited to map a limit cycle as a trajectory reduced in dimension and study the stability [7-8].
3. In a power system point of view, two cascaded tap changer network may form a basis for further research. Tap changers may be modeled in a more detailed manner than that considered in this paper. A constant power load (with dynamics) may be introduced to simulate more realistic behavior.

9. Acknowledgements

Fruitful discussions with Prof. I. A. Hiskens regarding tap changer transformer are acknowledged.

10. References

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