ECE 553, Spring '22 **FINAL EXAM INFORMATION** **Exam date: Wed May 11**

Procedure:

- You will be randomly assigned a topic from the list below. You will be given some time to prepare, after which you will present your topic to me. As follow-up questions, I can then ask you about any other topics from the list below, as well as any related questions about the material from Chapters 1–6 covered in class. You can take additional time to think about these follow-up questions. On average, be prepared to spend about 1–1.5 hours at the exam.
- This is an oral exam; you can prepare by sketching your answer on paper for your own reference, but you will need to present it to me by talking through it. For longer questions it is OK to just sketch the main steps initially, and I can then ask you to elaborate on some of them. You don't need to present things exactly as in the text, you can structure your presentation based on your preferences and understanding of the material.
- You should not bring anything with you to the exam room except a pen/pencil and blank paper. Access to class notes or any other resources is not allowed during the exam. However, I don't expect you to memorize long calculations or complicated formulas. Instead, I will focus on your conceptual understanding of the material.
- When you're ready, please use `this Google doc` to sign up for a specific time to come to the exam on Wednesday, May 11. There is no rush, as there will be enough time slots for everyone and I will keep the sign-up sheet open until the day before the exam. In case of any scheduling difficulties, please contact me right away.

Questions:

1. Definitions of first and second variation for a functional on a general (not necessarily finite-dimensional) vector space. Notion of a local minimum of such a functional. First-order necessary, second-order necessary, and second-order sufficient conditions for a local minimum.

2. Basic calculus of variations problem. Notions of weak and strong extrema. First-order necessary condition for a weak extremum (Euler-Lagrange equation). Two special cases ("no x" and "no y"), resulting integrals of motion.

3. Momentum and Hamiltonian. From Euler-Lagrange to Hamilton's canonical equations. Legendre transformation and its significance in calculus of variations. Principle of least action.

4. Variational problems with constraints. Necessary conditions for optimality in the presence of integral constraints and non-integral constraints.

5. Second-order necessary condition for a weak minimum (Legendre's condition).

6. Second-order sufficient conditions for a weak minimum (in terms of conjugate points).

7. Weierstrass-Erdmann corner conditions (direct derivation).

8. Weierstrass' excess function E and necessary condition for a strong minimum. Interpretation in terms of the Hamiltonian.

9. General formulation of the optimal control problem. Basic technical assumptions. Different forms of the cost functional and target set, passing from one to another via changes of variables.

10. First-order and second-order necessary conditions for the optimal control problem: the variational approach.

11. Maximum principle for the basic fixed-endpoint control problem. Main steps of the proof (just list the steps, you will then be asked to elaborate on one of them).

12. Maximum principle for the basic varying-endpoint control problem. Derivation of transversality condition.

13. Maximum principle for fixed-time problems, time-varying problems, and problems in Mayer form via changes of variables.

14. Bang-bang principle for linear systems (with respect to the time-optimal control problem). Nonlinear systems affine in controls, Lie brackets, and bang-bang vs. singular time-optimal controls.

15. Existence of optimal controls. Filippov's theorem and its application to Mayer problems and linear time-optimal problems.

16. Derivation of the HJB equation from the principle of optimality.

17. Sufficient conditions for optimality in terms of the HJB equation (finite-horizon case).

18. The concept of viscosity solution for PDEs. Value function as viscosity solution of the HJB equation.

19. Formulation of the finite-horizon LQR problem, derivation of the linear state feedback form of the optimal control using the maximum principle.

20. Derivation of the Riccati differential equation for the finite-horizon LQR problem. Verification of the optimal control law and value function using the HJB equation. Global existence of solution for the RDE.

21. Formulation and complete solution of the infinite-horizon, time-invariant LQR problem.

22. "Lucky question": present a topic of your choosing.