



Brief Paper

Supervision of integral-input-to-state stabilizing controllers[☆]João P. Hespanha^a, Daniel Liberzon^{b, *}, A. Stephen Morse^c^aDepartment of Electrical and Computer Engineering, University of California, Santa Barbara, CA 93106, USA^bCoordinated Science Laboratory, Univ. of Illinois at Urbana-Champaign, Urbana, IL 61801, USA^cDepartment of Electrical Engineering, Yale University, New Haven, CT 06520, USA

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Abstract

The subject of this paper is hybrid control of nonlinear systems with large-scale uncertainty. We describe a high-level controller, called a “supervisor”, which orchestrates logic-based switching among a family of candidate controllers. We show that in this framework, the problem of controller design at the lower level can be reduced to finding an integral-input-to-state stabilizing control law for an appropriate system with disturbance inputs. Employing the recently introduced “scale-independent hysteresis” switching logic, we prove that in the case of purely parametric uncertainty with unknown parameters taking values in a finite set the switching terminates in finite time and state regulation is achieved. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

This paper deals with the problem of controlling a nonlinear system in the presence of large modeling uncertainty. The basic idea behind the *supervisory control* approach to this problem is to employ logic-based switching among a suitably defined family of candidate controllers. The need for switching stems from the fact that typically no single controller would guarantee a desired behavior when connected with the poorly modeled process. Such switching schemes provide an alternative to more traditional continuously tuned adaptive control laws.

The main ingredients of the supervisory approach to controlling uncertain nonlinear systems are adopted from (Hespanha & Morse, 1999a) and have their roots in the work on supervisory control of uncertain linear systems reported in (Morse, 1996). In addition to the given process and the family of candidate controllers, the supervisory control system has three other subsystems, implemented

by the designer: the *multi-estimator*, the *monitoring signal generator*, and the *switching logic*. The task of the switching logic is to generate a *switching signal* which determines, at each instant of time, the candidate controller that is to be placed in the feedback loop. This controller selection is based on the values of the *monitoring signals*, which are obtained by taking appropriate integral norms of the *estimation errors* produced by the multi-estimator. Intuitively, the idea behind the switching strategy is to determine which of the monitoring signals is the smallest, and to choose the candidate controller that is designed for the respective parameter value. A form of hysteresis is used to slow the switching down. The resulting closed-loop system is therefore hybrid, as it combines discrete dynamics associated with the switching logic and continuous dynamics associated with the rest of the system.

We impose the condition that each candidate controller integral-input-to-state stabilizes the multi-estimator when the corresponding estimation error is viewed as an input. The concept of integral-input-to-state stability (iISS) was introduced in (Sontag, 1998). The iISS property is a variation of the more familiar input-to-state stability (ISS) property defined in (Sontag, 1989). Loosely speaking, the state of an input-to-state stable system is small if its inputs are small (“ L^∞ to L^∞ stability”), whereas the state of an integral-input-to-state stable system is small if its inputs have finite energy as defined by an appropriate integral

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(e.g., “ L^2 to L^∞ stability”). Every input-to-state stable system is integral-input-to-state stable, but the converse does not hold.

Requirements placed on the candidate controllers stem from the following considerations. It is desirable that for every frozen value of the switching signal, the closed-loop system be detectable through the corresponding estimation error. This detectability property, known as *tunability* (Morse, 1990), is crucial because adaptive tuning/switching algorithms are invariably designed with the goal of maintaining “smallness” of the estimation error associated with the controller that is currently in the feedback loop. It is established in (Hespanha & Morse, 1999a) that detectability, interpreted in a suitable sense for nonlinear systems, is guaranteed if the process is detectable and each candidate controller input-to-state stabilizes the multi-estimator with respect to the corresponding estimation error. That paper also shows how this result leads to a systematic technique for designing hybrid control laws that achieve global output regulation.

It turns out that in practical applications it is often difficult to achieve (or verify) input-to-state stability of the controller-estimator interconnection (Hespanha, Liberzon, & Morse, 1999; Chang, Hespanha, Morse, Netto, & Ortega, 2001). Thus it is of interest to weaken the requirements imposed on the candidate controllers, which is the primary motivation for the work described in this paper. In what follows, we show that the detectability property, interpreted in an integral sense, holds if the hypotheses are weakened by demanding iISS instead of ISS. We then consider the case of “exact matching”, in other words, we assume that there are no unmodeled dynamics, noise, or disturbances, so that the actual process is an unknown member of a finite family of admissible models. The scale-independent hysteresis switching logic introduced in (Hespanha, 1998; Hespanha & Morse, 1999b) guarantees that the switching stops in finite time and the continuous states of the supervisory control system converge to zero. A preliminary version of this result was stated in (Hespanha & Morse, 1999c).

The findings of this paper can thus be considered as integral versions of those previously reported in (Hespanha & Morse, 1999a). The use of iISS in the present context seems very natural because the monitoring signals are defined as appropriate integral norms of the estimation errors. Another contribution of this paper is that the analysis is carried out in the time domain and is more straightforward than that in (Hespanha & Morse, 1999a), where Lyapunov-like dissipation inequalities are used.

The concepts of integral-input-to-state stability and detectability that are exploited in the paper are formally defined and discussed in Section 2. The supervisory control architecture is presented in Section 3. In Section 4 we state the basic assumptions and establish the detectability result. Section 5 is devoted to the description of the switching logic and the analysis of the closed-loop system in the exact

matching case. The contributions of the paper are briefly summarized and assessed in Section 6.

2. Integral-input-to-state stability and detectability

First, recall that a function $\alpha : [0, \infty) \rightarrow [0, \infty)$ is said to be of class \mathcal{K} if it is continuous, strictly increasing, and $\alpha(0) = 0$. If α is also unbounded, then it is said to be of class \mathcal{K}_∞ . A function $\beta : [0, \infty) \times [0, \infty) \rightarrow [0, \infty)$ is said to be of class \mathcal{KL} if $\beta(\cdot, t)$ is of class \mathcal{K} for each fixed $t \geq 0$ and $\beta(r, t)$ decreases to 0 as $t \rightarrow \infty$ for each fixed $r \geq 0$. We will write $\alpha \in \mathcal{K}_\infty$, $\beta \in \mathcal{KL}$ when α is a class \mathcal{K}_∞ function and β is a class \mathcal{KL} function, respectively.

Consider the general nonlinear system

$$\begin{aligned} \dot{x} &= f(x, d), \\ y &= h(x), \end{aligned} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state, $d \in \mathbb{R}^k$ is the locally essentially bounded (disturbance) input, and $y \in \mathbb{R}^l$ is the output. In this paper, we only consider smooth systems for simplicity (this requirement can be significantly relaxed, as discussed in the references cited below). Following (Sontag & Wang, 1997), we will say that the system (1) is *input/output-to-state stable* (IOSS) if for some functions $\gamma_1, \gamma_2 \in \mathcal{K}_\infty$ and $\beta \in \mathcal{KL}$, for every initial state $x(0)$, and every input d the corresponding solution of (1) satisfies the inequality

$$|x(t)| \leq \beta(|x(0)|, t) + \gamma_1(\|d\|_{[0,t]}) + \gamma_2(\|y\|_{[0,t]}) \quad (2)$$

as long as it is defined. Here $\|\cdot\|_{[a,b]}$ stands for the (essential) supremum norm of a signal restricted to the interval $[a, b]$. We will sometimes omit the subscripts if clear from the context.

IOSS represents a natural detectability property of the system, which basically says that the state eventually becomes small if the inputs and outputs are small. For this reason, we will sometimes use the term “detectability” when referring to this property; however, the precise notion being used is IOSS, which is not to be confused with other possible interpretations of detectability for nonlinear systems. Removing the γ_2 term from (2), one recovers the standard notion of *input-to-state stability* (ISS) with respect to d , as defined in (Sontag, 1989). Although in this paper we take the equilibrium state of the system (1) to be the origin, the subsequent definitions and results can be extended to the case of nonzero equilibrium states (see Hespanha & Morse, 1999a,c).

An integral variant of the above detectability notion can be defined as follows (Angeli, Sontag, & Wang, 2000). We will say that the system (1) is *integral input/output-to-state stable* (iIOSS) if for some functions $\alpha_0, \gamma_1, \gamma_2 \in \mathcal{K}_\infty$ and $\beta \in \mathcal{KL}$, for every initial state $x(0)$, and every input d

the corresponding solution of (1) satisfies the inequality

$$\alpha_0(|x(t)|) \leq \beta(|x(0)|, t) + \int_0^t \gamma_1(|d(s)|) ds + \int_0^t \gamma_2(|y(s)|) ds \tag{3}$$

as long as it is defined. The corresponding property for systems without inputs, obtained by removing the γ_1 term from (3), is called *integral-output-to-state stability* (iOSS). Similarly, removing the γ_2 term from (3), we arrive at *integral-input-to-state stability* (iISS), a property introduced in (Sontag, 1998). We will need the following simple fact (cf. Sontag, 1998, Proposition 6).

Lemma 1. *Suppose that the system (1) is iIOSS. Suppose that the initial state $x(0)$ and the disturbance d are such that the corresponding solution of (1) is globally defined, $\int_0^\infty \gamma_1(|d(s)|) ds < \infty$, and $\int_0^\infty \gamma_2(|y(s)|) ds < \infty$. Then we have $x(t) \rightarrow 0$ as $t \rightarrow \infty$.*

We will also need the following technical lemma. It states that IOSS implies the “mixed” detectability property, where one takes a supremum norm of some inputs and outputs and an integral norm of others. This lemma is proved by standard arguments using the characterization of IOSS in terms of an “exponential-decay IOSS-Lyapunov function” (Sontag & Wang, 1997).

Lemma 2. *Assume that the system (1) is IOSS. Suppose that an arbitrary partition of its inputs and outputs into two groups, denoted by v_1 and v_2 , is given (here v_1 and v_2 are vectors, each of which is allowed to contain both input and output components). Then there exist functions $\bar{\alpha}_0, \bar{\gamma}_1, \bar{\gamma}_2 \in \mathcal{K}_\infty$ and $\bar{\beta} \in \mathcal{KL}$ such that for every initial state $x(0)$ and every input d the solution of (1) satisfies the following inequality as long as it exists:*

$$\bar{\alpha}_0(|x(t)|) \leq \bar{\beta}(|x(0)|, t) + \bar{\gamma}_1(\|v_1\|_{[0,t]}) + \int_0^t \bar{\gamma}_2(|v_2(s)|) ds. \tag{4}$$

3. Supervisory control system architecture

Let \mathbb{P} be an unknown process, with dynamics of the form

$$\dot{x} = f(x, u),$$

$$y = h(x),$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, and $y \in \mathbb{R}^l$ is the measured output. We assume that \mathbb{P} is a member of some family of systems $\bigcup_{p \in \mathcal{P}} \mathcal{F}_p$, where \mathcal{P} is an index set. For each $p \in \mathcal{P}$, the subfamily \mathcal{F}_p can be thought of as consisting of a *nominal process model* \mathbb{M}_p together with a collection of its “perturbed” versions. However, the developments of this section and the next one do not rely on any special structure of the \mathcal{F}_p . The only explicit assumption that we impose on the unknown process is detectability.

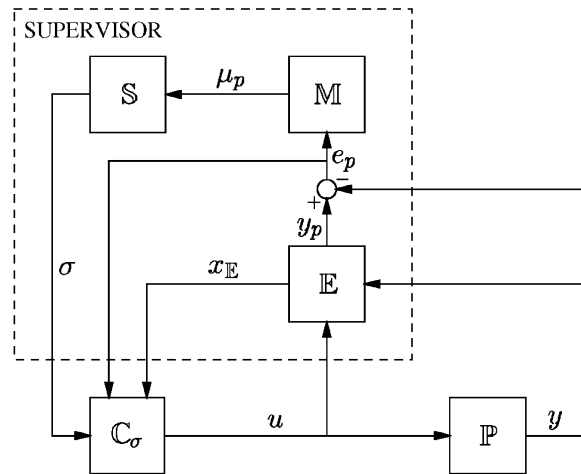


Fig. 1. Supervisory control architecture.

Assumption 1. The process \mathbb{P} is IOSS.

The problem of interest is to design a feedback control law that drives the state x (or at least the output y) of \mathbb{P} to zero.¹ To solve this problem, we will develop a “high-level” controller, called a *supervisor*, whose purpose is to orchestrate the switching among a suitably defined family of *candidate controllers* $\mathbb{C}_p, p \in \mathcal{P}$. It is convenient to think of each \mathbb{C}_p as a controller that would be used to solve the regulation problem if \mathbb{P} were known to be a member of \mathcal{F}_p . For example, if for each $p \in \mathcal{P}$ the candidate controller \mathbb{C}_p takes the form

$$\begin{aligned} \dot{x}_C &= g_p(x_C, y), \\ u_p &= r_p(x_C, y), \end{aligned} \tag{5}$$

then the switched controller is given by

$$\begin{aligned} \dot{x}_C &= g_\sigma(x_C, y), \\ u &= r_\sigma(x_C, y), \end{aligned}$$

where σ is a piecewise constant *switching signal* taking values in \mathcal{P} . Accordingly, the switched controller is denoted by \mathbb{C}_σ . We will actually use a somewhat more general form for the candidate controllers, described below.

The supervisor consists of three subsystems (see Fig. 1):

Multi-estimator \mathbb{E} —a dynamical system whose inputs are the process’ input u and output y , whose state is denoted by x_E , and whose outputs are denoted by $y_p, p \in \mathcal{P}$.

Monitoring signal generator \mathbb{M} —a dynamical system whose inputs are the *estimation errors*

$$e_p := y_p - y, \quad p \in \mathcal{P} \tag{6}$$

and whose outputs $\mu_p, p \in \mathcal{P}$ are suitably defined integral norms of the estimation errors, called *monitoring signals*.

¹ More general problems of regulation about nonzero equilibrium states can be treated similarly (cf. Hespanha & Morse, 1999a,c).

Switching logic \mathbb{S} —a dynamical system whose inputs are the monitoring signals μ_p , $p \in \mathcal{P}$ and whose output is the *switching signal* σ .

We write the multi-estimator \mathbb{E} as

$$\begin{aligned}\dot{x}_{\mathbb{E}} &= F(x_{\mathbb{E}}, y, u) \\ y_p &= h_p(x_{\mathbb{E}}), \quad p \in \mathcal{P},\end{aligned}\quad (7)$$

where we assume that $h_p(0) = 0$ for each $p \in \mathcal{P}$. The understanding is that y_p would converge to the process output y asymptotically if \mathbb{M}_p were the actual process model and there were no noise or disturbances (cf. Section 5). If \mathcal{P} is a finite set, \mathbb{E} can be realized simply as a parallel connection of individual estimator equations of the form

$$\begin{aligned}\dot{x}_p &= f_p(x_p, y, u), \\ y_p &= \tilde{h}_p(x_p)\end{aligned}$$

for $p \in \mathcal{P}$, in which case one has $x_{\mathbb{E}} = \text{stack}\{x_p, p \in \mathcal{P}\}$. If the set \mathcal{P} is infinite, the multi-estimator can often be implemented using the idea of *state-sharing* (Hespanha & Morse, 1999a; Morse, 1996). This technique in fact leads to a more efficient way of designing a supervisory control system even for the case when \mathcal{P} has a finite but large number of elements.

We write the switched controller \mathbb{C}_σ as

$$\begin{aligned}\dot{x}_{\mathbb{C}} &= g_\sigma(x_{\mathbb{C}}, x_{\mathbb{E}}, e_\sigma), \\ u &= r_\sigma(x_{\mathbb{C}}, x_{\mathbb{E}}, e_\sigma)\end{aligned}\quad (8)$$

with $r_p(0, 0, 0) = 0 \forall p \in \mathcal{P}$. For each frozen value $\sigma \equiv p \in \mathcal{P}$, the above equations model the candidate controller \mathbb{C}_p . Note that here we are assuming the entire state $x_{\mathbb{E}}$ of the multi-estimator \mathbb{E} to be available for control, which is reasonable because \mathbb{E} is implemented by the control designer. Since $y = h_p(x_{\mathbb{E}}) - e_p$ for each $p \in \mathcal{P}$ by virtue of (6) and (7), this set-up includes static output feedback $u_p = k_p(y)$ and dynamic output feedback (5) as special cases. This explains why we did not include y explicitly as an input to \mathbb{C}_σ . A particular choice of inputs to \mathbb{C}_σ may vary.

The underlying decision-making strategy used by the supervisor basically consists in selecting for σ , from time to time, the candidate controller index q whose corresponding monitoring signal μ_q is currently the smallest. Intuitively, the motivation for doing this is that the nominal process model with the smallest monitoring signal “best” approximates the actual process, and thus the candidate controller associated with that model can be expected to do the best job of controlling the process. This idea originates in the concept of certainty equivalence from parameter adaptive control. A justification for such a strategy will be seen more clearly in light of the results to follow. Precise definitions of the monitoring signal generator and the switching logic are deferred to Section 5.

4. Detectability

For an arbitrary fixed $q \in \mathcal{P}$ we can use the formula (6) to rewrite the multi-estimator (7) as

$$\dot{x}_{\mathbb{E}} = F(x_{\mathbb{E}}, h_q(x_{\mathbb{E}}) - e_q, u) := \bar{F}_q(x_{\mathbb{E}}, e_q, u). \quad (9)$$

Consider the following auxiliary system, which we call the *injected system*:

$$\begin{aligned}\dot{\bar{x}}_E &= \bar{F}_q(\bar{x}_E, d, r_q(\bar{x}_{\mathbb{C}}, \bar{x}_E, d)), \\ \dot{\bar{x}}_C &= g_q(\bar{x}_{\mathbb{C}}, \bar{x}_E, d).\end{aligned}\quad (10)$$

Here d is a fictitious disturbance input. If $d(\cdot) = e_q(\cdot)$, the system (10) has the same solutions as the one that results when the q th candidate controller \mathbb{C}_q , given by (8) with $\sigma \equiv q$, is connected to the multi-estimator given by (9). Our choice of candidate controllers is based on the following assumption.

Assumption 2. For each $q \in \mathcal{P}$ the injected system (10) is iISS with respect to the disturbance d .

This means that for each $q \in \mathcal{P}$ there exist class \mathcal{K}_∞ functions α_q and γ_q and a class \mathcal{KL} function β_q such that for all initial states $\bar{x}_E(0)$, $\bar{x}_{\mathbb{C}}(0)$ and all locally essentially bounded $d(\cdot)$ the solution of the system (10) satisfies the following inequality:

$$\begin{aligned}\alpha_q \left(\left| \begin{pmatrix} \bar{x}_E(t) \\ \bar{x}_{\mathbb{C}}(t) \end{pmatrix} \right| \right) &\leq \beta_q \left(\left| \begin{pmatrix} \bar{x}_E(0) \\ \bar{x}_{\mathbb{C}}(0) \end{pmatrix} \right|, t \right) \\ &+ \int_0^t \gamma_q(|d(s)|) ds \quad \forall t \geq 0.\end{aligned}\quad (11)$$

For linear systems, this property automatically follows from (internal) stability of the injected system, but for nonlinear systems this is not the case. The problem of designing integral-input-to-state stabilizing control laws for nonlinear systems is addressed in (Liberzon, Sontag, & Wang, 1999; Liberzon, 1999; Teel & Praly, 2000; Liberzon, Sontag, & Wang, 2001). Note that in the present context the disturbance d will be identified with the estimation error e_q , which is an input to the q th controller. This substantially simplifies the control design problem because, as explained in the above references, it is in general easier to find a control law of the form $u = k(x, d)$ than one of the form $u = k(x)$.

We now turn our attention to the system that results when the q th candidate controller \mathbb{C}_q is placed in the feedback loop with the process \mathbb{P} and the multi-estimator \mathbb{E} . The dynamics of this system are described by the following equations:

$$\begin{aligned}\dot{x} &= f(x, r_q(x_{\mathbb{C}}, x_{\mathbb{E}}, h_q(x_{\mathbb{E}}) - h(x))), \\ \dot{x}_{\mathbb{E}} &= F(x_{\mathbb{E}}, h(x), r_q(x_{\mathbb{C}}, x_{\mathbb{E}}, h_q(x_{\mathbb{E}}) - h(x))), \\ \dot{x}_{\mathbb{C}} &= g_q(x_{\mathbb{C}}, x_{\mathbb{E}}, h_q(x_{\mathbb{E}}) - h(x)).\end{aligned}\quad (12)$$

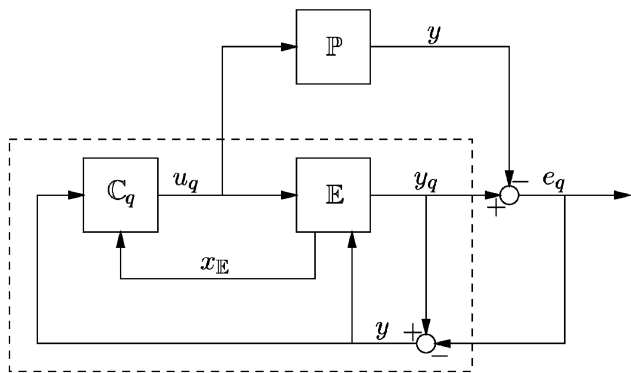


Fig. 2. The closed-loop system (12).

Let us denote the state $(x', x'_E, x'_C)'$ of this system by \mathbf{x} and regard the estimation error $e_q = h_q(x_E) - h(x)$ as the output of the system. Our main result in this section states that the above system is iOSS through e_q . This means, loosely speaking, that the state \mathbf{x} of (12) can be bounded by a suitable integral norm of e_q (see Section 2 for the precise definition). Therefore, such a result provides a justification for switching control strategies of the kind considered in this paper, which are based on choosing a candidate controller whose index minimizes some integral norm of the estimation error (see Section 5).

Theorem 3. *Let Assumptions 1 and 2 hold. For each fixed $q \in \mathcal{P}$, the system (12) is iOSS with respect to e_q , i.e., along its solutions we have*

$$\alpha_0(|\mathbf{x}(t)|) \leq \beta_0(|\mathbf{x}(0)|, t) + \int_0^t \tilde{\gamma}_q(|e_q(s)|) ds \quad (13)$$

for some functions $\alpha_0, \tilde{\gamma}_q \in \mathcal{K}_\infty$ and $\beta_0 \in \mathcal{K}\mathcal{L}$.

Proof. Fix an arbitrary $q \in \mathcal{P}$. A convenient way of thinking about the system (12) is facilitated by the block diagram shown in Fig. 2, where the system enclosed in the dashed box becomes equivalent to the injected system (10) upon setting $d = e_q$.

Let $x_{\text{EC}} := (x'_E, x'_C)'$. In view of Assumption 2, we have

$$\alpha_q(|x_{\text{EC}}(t)|) \leq \beta_q(|x_{\text{EC}}(0)|, t) + \int_0^t \gamma_q(|e_q(s)|) ds, \quad (14)$$

where $\alpha_q, \gamma_q \in \mathcal{K}_\infty$ and $\beta_q \in \mathcal{K}\mathcal{L}$. As for the process, Assumption 1 and the formula (6) give

$$\begin{aligned} |x(t)| &\leq \beta(|x(0)|, t) + \gamma_1(\|u_q\|) + \gamma_2(\|y\|) \\ &\leq \beta(|x(0)|, t) + \gamma_1(\|u_q\|) + \gamma_2(\|2y_q\|) + \gamma_2(\|2e_q\|), \end{aligned}$$

where $\gamma_1, \gamma_2 \in \mathcal{K}_\infty$ and $\beta \in \mathcal{K}\mathcal{L}$. Recall that we have $u_q = r_q(x_C, x_E, e_q)$ with $r_q(0, 0, 0) = 0$ and $y_q = h_q(x_E)$ with $h_q(0) = 0$. In view of this, it is easy to check that for a suitable class \mathcal{K}_∞ function γ we can rewrite the above inequality as

$$|x(t)| \leq \beta(|x(0)|, t) + \gamma(\|(x'_E, x'_C, e'_q)'\|).$$

This implies that the subsystem corresponding to \mathbb{P} , when viewed as a system with inputs x_E, x_C, e_q and state x , is ISS. Applying Lemma 2 with $v_1 := x_{\text{EC}}$ and $v_2 := e_q$ to this system, we conclude that there exist functions $\bar{\alpha}_0, \bar{\gamma}_1, \bar{\gamma}_2 \in \mathcal{K}_\infty$ and $\bar{\beta} \in \mathcal{K}\mathcal{L}$ such that we have

$$\bar{\alpha}_0(|x(t)|) \leq \bar{\beta}(|x(0)|, t) + \bar{\gamma}_1(\|x_{\text{EC}}\|) + \int_0^t \bar{\gamma}_2(|e_q(s)|) ds. \quad (15)$$

The calculations performed below are quite similar to the ones used in (Sontag, 1989) for the analysis of cascade systems. We employ the trick of dividing a time interval under consideration into two parts, exploiting the fact that in the definitions of Section 2 one could replace 0 by an arbitrary initial time $t_0 \geq 0$ (because the systems under consideration are time-invariant). We obtain

$$\begin{aligned} \bar{\alpha}_0(|x(t)|) &\leq \bar{\beta}(|x(t/2)|, t/2) \\ &\quad + \bar{\gamma}_1(\|x_{\text{EC}}\|_{[t/2, t]}) + \int_{t/2}^t \bar{\gamma}_2(|e_q(s)|) ds. \end{aligned} \quad (16)$$

In view of (15) and (14), it is straightforward (although tedious) to check that the first term on the right-hand side of (16) is bounded by

$$\beta_1(|x(0)|, t) + \beta_2(|x_{\text{EC}}(0)|, t) + \hat{\alpha}_1 \left(\int_0^{t/2} \hat{\gamma}_1(|e_q(s)|) ds \right),$$

where the functions $\beta_1, \beta_2 \in \mathcal{K}\mathcal{L}$ are defined by

$$\begin{aligned} \beta_1(r, t) &:= \bar{\beta}(4\bar{\alpha}_0^{-1}(4\bar{\beta}(r, t/2)), t/2), \\ \beta_2(r, t) &:= \bar{\beta}(4\bar{\alpha}_0^{-1}(4\bar{\gamma}_1(2\alpha_q^{-1}(2\beta_q(r, 0)))), t/2) \end{aligned}$$

and the functions $\hat{\alpha}_1, \hat{\gamma}_1 \in \mathcal{K}_\infty$ are defined by

$$\begin{aligned} \hat{\alpha}_1(r) &:= \max\{\bar{\beta}(4\bar{\alpha}_0^{-1}(4\bar{\gamma}_1(2\alpha_q^{-1}(2r))), \bar{\beta}(4\bar{\alpha}_0^{-1}(4r))\}, \\ \hat{\gamma}_1(r) &:= \max\{\gamma_q(r), \bar{\gamma}_2(r)\}. \end{aligned}$$

Similarly, the second term on the right-hand side of (16) is bounded by

$$\beta_3(|x_{\text{EC}}(0)|, t) + \hat{\alpha}_2 \left(\int_0^t \gamma_q(|e_q(s)|) ds \right),$$

where the functions $\beta_3 \in \mathcal{K}\mathcal{L}$ and $\hat{\alpha}_2 \in \mathcal{K}_\infty$ are defined by

$$\beta_3(r, t) := \bar{\gamma}_1(2\alpha_q^{-1}(2\beta_q(r, t/2))), \quad \hat{\alpha}_2(r) := \bar{\gamma}_1(2\alpha_q^{-1}(2r)).$$

Combining the above inequalities, we see that (16) reduces to

$$\begin{aligned} \bar{\alpha}_0(|x(t)|) &\leq \beta_1(|x(0)|, t) + \beta_2(|x_{\text{EC}}(0)|, t) \\ &\quad + \beta_3(|x_{\text{EC}}(0)|, t) + \hat{\alpha} \left(\int_0^t \tilde{\gamma}_q(|e_q(s)|) ds \right), \end{aligned}$$

where

$$\hat{\alpha}(r) := \max\{\hat{\alpha}_1(r), r\} + \hat{\alpha}_2(r), \quad \tilde{\gamma}_q(r) := \max\{\hat{\gamma}_1(r), \gamma_q(r)\}.$$

Therefore, we have

$$\hat{\alpha}_0(|x(t)|) \leq \hat{\alpha}^{-1}(\beta_1(|x(0)|, t) + \beta_2(|x_{\text{EC}}(0)|, t) + \beta_3(|x_{\text{EC}}(0)|, t)) + \int_0^t \tilde{\gamma}_q(|e_q(s)|) ds, \tag{17}$$

where $\hat{\alpha}_0(r) := \hat{\alpha}^{-1}(\bar{\alpha}_0(r)/2)$. The formulas (14) and (17) yield (13) with

$$\alpha_0(r) := \min\{\alpha_q(r/2), \hat{\alpha}_0(r/2)\},$$

$$\beta_0(r, t) := \beta_q(r, t) + \hat{\alpha}^{-1}(\beta_1(r, t) + \beta_2(r, t) + \beta_3(r, t)). \quad \square$$

Theorem 3 is quite general in the sense that it does not rely on any explicit assumptions regarding the process uncertainty. One could also consider the situation where the process has, in addition to the control input u , a disturbance input w , which enters in such a way that Assumption 1 is still satisfied (Hespanha and Morse, 1999a,c). In this case, the right-hand side of the Eq. (13) would contain one additional term of the form $\hat{\gamma}(\|w\|_{[0,t]})$, where $\hat{\gamma} \in \mathcal{K}_\infty$.

5. Exact matching

The goal of this section is to address the problem of global state or output regulation by hybrid output feedback. We do this for the special case when the process \mathbb{P} to be controlled takes the form

$$\begin{aligned} \dot{x} &= f(x, u, p^*), \\ y &= h(x), \end{aligned} \tag{18}$$

where f and h are known functions and p^* is an unknown element of a finite index set \mathcal{P} . Thus we assume that there are no unmodeled dynamics, noise, or disturbances, so that the unknown process \mathbb{P} exactly matches one of a finite number of nominal process models. Assumption 3 basically demands that the estimation error associated with the true parameter value be small in an integral sense.

Assumption 3. There exists a positive number λ with the property that for arbitrary initial conditions $x(0)$, $x_E(0)$, $x_C(0)$ there exists a constant C such that we have $\int_0^t e^{\lambda s} \tilde{\gamma}_{p^*}(|e_{p^*}(s)|) ds \leq C$ for all t that belong to the maximal interval on which the solution of the system is defined. Here $\tilde{\gamma}_{p^*}$ is the function appearing in the formula (13) which expresses the statement of Theorem 3, with $q = p^*$.

It is not hard to see that in the case when $\tilde{\gamma}_{p^*}$ is locally Lipschitz, the integral $\int_0^t e^{\lambda s} \tilde{\gamma}_{p^*}(|e_{p^*}(s)|) ds$ is bounded if e_{p^*} and $\int_0^t e^{\lambda s} |e_{p^*}(s)| ds$ are bounded. Then Assumption 3 is satisfied with λ small enough if the multi-estimator is designed so that e_{p^*} converges to zero exponentially for every control signal u . Several examples of such multi-estimator design for nonlinear systems can be found in (Hespanha, 1998).

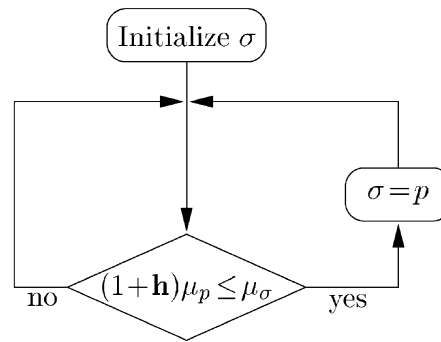


Fig. 3. Switching logic.

We generate the monitoring signals using the differential equations

$$\dot{\mu}_p = -\lambda \mu_p + \tilde{\gamma}_p(|e_p|), \quad p \in \mathcal{P} \tag{19}$$

with the same $\lambda > 0$ as in Assumption 3 and with initial values satisfying $\mu_p(0) > 0$. As for the switching logic, we consider the *scale-independent hysteresis switching logic* proposed in (Hespanha, 1998; Hespanha & Morse, 1999b). Let us pick a number $h > 0$ called the *hysteresis constant*. The functioning of the switching logic is as follows (see Fig. 3). First, we set $\sigma(0) = \arg \min_{p \in \mathcal{P}} \{\mu_p(0)\}$. Suppose that at a certain time t_i the value of σ has just switched to some $q \in \mathcal{P}$. We then keep σ fixed until a time $t_{i+1} > t_i$ such that $(1 + h) \min_{p \in \mathcal{P}} \{\mu_p(t_{i+1})\} \leq \mu_q(t_{i+1})$, at which point we set $\sigma(t_{i+1}) = \arg \min_{p \in \mathcal{P}} \{\mu_p(t_{i+1})\}$. (When the indicated minimum is not unique, a particular value for σ among those that achieve the minimum can be chosen arbitrarily.) Repeating this procedure, we generate a piecewise constant signal σ that is continuous from the right everywhere. As we will see, by setting $\mu_p(0) > 0$ for all $p \in \mathcal{P}$ we avoid chattering.

The overall supervisory control system is a hybrid system with continuous states $\mathbf{x} = (x', x'_E, x'_C)'$ and μ_p , $p \in \mathcal{P}$ and discrete state σ . The following is a corollary of Theorem 3 and the results of (Hespanha, 1998; Hespanha & Morse, 1999b).

Corollary 4. Let \mathcal{P} be a finite set, and consider the supervisory control system defined by (6), (7), (8), (18), (19), and the switching logic described above, with arbitrary initial conditions satisfying $\mu_p(0) > 0$ for all $p \in \mathcal{P}$. Under Assumptions 1, 2, and 3, there exists a time T^* such that $\sigma(t) = q^* \in \mathcal{P}$ for all $t \geq T^*$, i.e., the switching stops in finite time, and all the continuous states converge to 0 as $t \rightarrow \infty$.

Proof. Let us define (for analysis purposes only) the scaled monitoring signals

$$\bar{\mu}_p(t) := e^{\lambda t} \mu_p(t), \quad p \in \mathcal{P}. \tag{20}$$

In view of (19) we have

$$\bar{\mu}_p(t) = \bar{\mu}_p(0) + \int_0^t e^{\lambda s} \tilde{\gamma}_p(|e_p(s)|) ds, \quad p \in \mathcal{P}. \quad (21)$$

The scale independence property of the switching logic implies that replacing μ_p by $\bar{\mu}_p$ would have no effect on σ (Hespanha, 1998; Hespanha & Morse, 1999b; Morse, 1996). From (21) we see that each $\bar{\mu}_p$ is nondecreasing. This, the finiteness of \mathcal{P} , and the fact that $\bar{\mu}_p(0) > 0$ for each $p \in \mathcal{P}$ guarantee the existence of a positive number ε such that $\bar{\mu}_p(t) > \varepsilon$ for all $t \geq 0$ and all $p \in \mathcal{P}$. It is not hard to conclude now from the definition of the switching logic that chattering cannot occur; in fact, there must be an interval $[0, T)$ of maximal length on which the solution of the system is defined, and σ can only have a finite number of discontinuities on each proper subinterval of $[0, T)$. For details, see (Hespanha, 1998; Hespanha & Morse, 1999b).

Observe that $\bar{\mu}_{p^*}$ is bounded on $[0, T)$ by virtue of Assumption 3. It follows that the signals $\bar{\mu}_p$ satisfy the hypotheses of the Scale-Independent Hysteresis Switching Lemma (Hespanha, 1998; Hespanha & Morse, 1999b) which enables us to conclude that the switching stops in finite time. More precisely, there exists a time $T^* < T$ such that $\sigma(t) = q^* \in \mathcal{P}$ for all $t \geq T^*$. In addition, $\bar{\mu}_{q^*}$ is bounded on $[0, T)$. Using (21) with $p = q^*$ and the boundedness of $\bar{\mu}_{q^*}$, we see that the integral $\int_0^T \tilde{\gamma}_{q^*}(|e_{q^*}(s)|) ds$ is finite (recall that λ is positive). In view of (13) this implies that x , x_E , and x_C are bounded on $[0, T)$. The estimation error $e_p = y_p - y$ is then also bounded for each $p \in \mathcal{P}$, hence each $\tilde{\gamma}_p(|e_p|)$ is bounded in view of the continuity of $\tilde{\gamma}_p$. Furthermore, all the monitoring signals μ_p remain bounded because they are generated by the system (19) with bounded inputs $\tilde{\gamma}_p(|e_p|)$. Thus we see that $T = \infty$, i.e., the solution of the system is globally defined. Having established that $T = \infty$, we can apply Lemma 1 to the overall system, which enables us to conclude that $\mathbf{x} = (x', x'_E, x'_C)'$, and consequently $\mu_p, p \in \mathcal{P}$, converge to 0 as $t \rightarrow \infty$. \square

As seen from the proof of Theorem 3, the function $\tilde{\gamma}_q$ appearing in the formula (13) depends on the functions that express the IOSS property of the uncertain process \mathbb{P} . Therefore, it is rather restrictive to assume the knowledge of $\tilde{\gamma}_q$ or its upper bound for each $q \in \mathcal{P}$. An alternative construction presented below allows us to work directly with the functions γ_q from (11), but requires a somewhat different convergence proof. Let us replace Assumption 3 by the following.

Assumption 3'. There exists a positive number λ with the property that for arbitrary initial conditions $x(0), x_E(0), x_C(0)$ there exist constants C_1, C_2 such that we have $|e_{p^*}(t)| \leq C_1$ and $\int_0^t e^{\lambda s} \gamma_{p^*}(|e_{p^*}(s)|) ds \leq C_2$ for all t that belong to the maximal interval on which the solution of the system is defined. Here γ_{p^*} is the function appearing in the formula (11) associated with Assumption 2, for $q = p^*$.

Instead of using the Eq. (19), we now generate the monitoring signals by

$$\dot{\mu}_p = -\lambda \mu_p + \gamma_p(|e_p|), \quad p \in \mathcal{P} \quad (22)$$

with the same $\lambda > 0$ as in Assumption 3' and with initial values satisfying $\mu_p(0) > 0$. We use the same scale-independent hysteresis switching logic as before. Then the following result holds.

Proposition 5. Let \mathcal{P} be a finite set, and consider the supervisory control system defined by (6), (7), (8), (18), (22), and the switching logic described above, with arbitrary initial conditions satisfying $\mu_p(0) > 0$ for all $p \in \mathcal{P}$. Under Assumptions 1, 2, and 3', there exists a time T^* such that $\sigma(t) = q^* \in \mathcal{P}$ for all $t \geq T^*$, i.e., the switching stops in finite time, and all the continuous states converge to 0 as $t \rightarrow \infty$.

Proof. Exactly as in the proof of Corollary 4, using the scaled monitoring signals (20), we prove that there exists a time $T^* < T$ such that $\sigma(t) = q^* \in \mathcal{P}$ for all $t \geq T^*$, and that the integral $\int_0^T \gamma_{q^*}(|e_{q^*}(s)|) ds$ is finite. In view of (11) this implies that x_E and x_C are bounded on $[0, T)$ because solutions of the multi-estimator (7) with the q^* -th candidate controller in the feedback loop coincide with solutions of the injected system (10) when $q = q^*$ and $d(\cdot) = e_{q^*}(\cdot)$. Since e_{p^*} is bounded by Assumption 3, it follows that $y = y_{p^*} - e_{p^*}$ remains bounded as well. Therefore, $e_p = y_p - y$ is bounded for each $p \in \mathcal{P}$. Each $\gamma_p(|e_p|)$ is then also bounded in view of the continuity of γ_p . Furthermore, all the monitoring signals μ_p remain bounded because they are generated by (19) with bounded inputs $\gamma_p(|e_p|)$. Finally, x is bounded in view of Assumption 1. Thus we see that $T = \infty$, i.e., the solution of the system is globally defined.

Having established that $T = \infty$, we can apply Lemma 1 to the injected system (10), which enables us to conclude that $x_E, x_C \rightarrow 0$ as $t \rightarrow \infty$. Since all signals are bounded, the derivative $\dot{e}_{q^*} = \dot{y}_{q^*} - \dot{y}$ is bounded. The boundedness of e_{q^*}, \dot{e}_{q^*} , and of the integral $\int_0^\infty \gamma_{q^*}(|e_{q^*}(s)|) ds$ is well known to imply that $e_{q^*} \rightarrow 0$ as $t \rightarrow \infty$ (see, e.g., Aizerman & Gantmacher, 1964, p. 58). Thus we have $y = y_{q^*} - e_{q^*} \rightarrow 0$, hence $x \rightarrow 0$ by Assumption 1, and $\mu_p \rightarrow 0$ for each $p \in \mathcal{P}$ as before. \square

If one is only concerned with output regulation and not state regulation, a close examination of the proof of Proposition 5 reveals that the assumptions can be weakened even further.

Assumption 1'. The state x of \mathbb{P} is bounded if the control input u and the output y are bounded.

Assumption 2'. For each $q \in \mathcal{P}$ and every disturbance d satisfying $\int_0^\infty \gamma_q(|d(s)|) ds < \infty$, the solution (\bar{x}_E, \bar{x}_C) of the injected system (10) remains bounded for arbitrary initial conditions and we have $h_q(\bar{x}_E) \rightarrow 0$.

Proposition 6. Let \mathcal{P} be a finite set. Under Assumptions 1', 2', and 3', all the signals in the supervisory control system defined by (6), (7), (8), (18), (22), and the switching logic described above remain bounded for arbitrary initial conditions satisfying $\mu_p(0) > 0$ for all $p \in \mathcal{P}$. There exists a time T^* such that $\sigma(t) = q^* \in \mathcal{P}$ for all $t \geq T^*$, i.e., the switching stops in finite time, and we have $y(t) \rightarrow 0$ as $t \rightarrow \infty$.

A specific supervisory control system satisfying Assumption 2' was studied in (Hespanha et al., 1999).

6. Concluding remarks

In this paper, we have described a framework for supervisory control of poorly modeled nonlinear systems. By using the integral versions of input-to-state stability and detectability, we were able to weaken the assumptions imposed in earlier work (Hespanha & Morse, 1999a). The analysis techniques presented here are more direct than, and provide an alternative to, the methods used in (Hespanha & Morse, 1999a). Our results underscore the importance of developing systematic methods for designing integral-input-to-state stabilizing controllers for general classes of nonlinear systems; see (Liberzon et al., 1999; Liberzon, 1999; Teel & Praly, 2000; Liberzon et al., 2001) for more information on this topic.

If the injected system (10) is iISS but not ISS, then it may happen that its state converges to zero when the input d has a finite energy expressed by a suitable integral, but blows up under the action of a bounded input. Thus the supervisory control algorithms presented here might be less robust with respect to bounded noise, disturbances, and unmodeled dynamics than the one considered in (Hespanha & Morse, 1999a). This constitutes an important area for further investigation.

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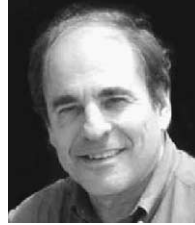
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