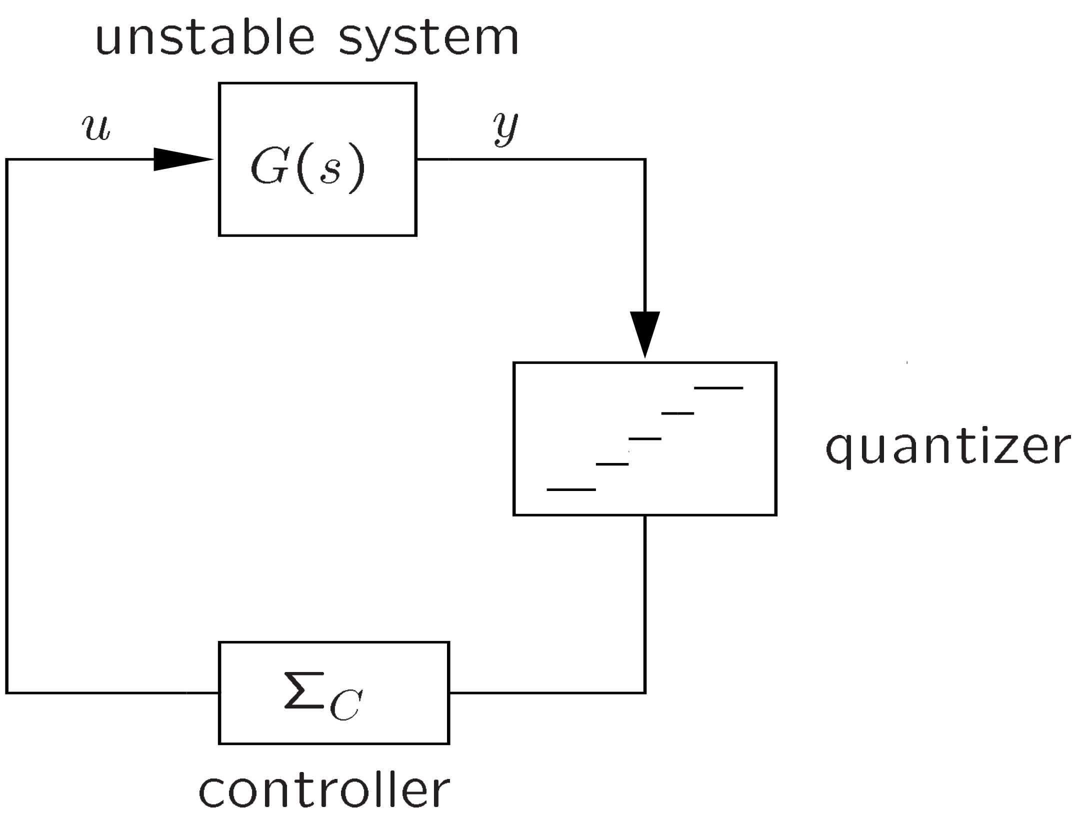
Quantized feedback stabilization of linear systems

Roger W. Brockett
Harvard University

Daniel Liberzon
Yale University

BASIC PROBLEM

Stabilize an LTI system using quantized measurements.



Previous work:

R. Curry (1970)

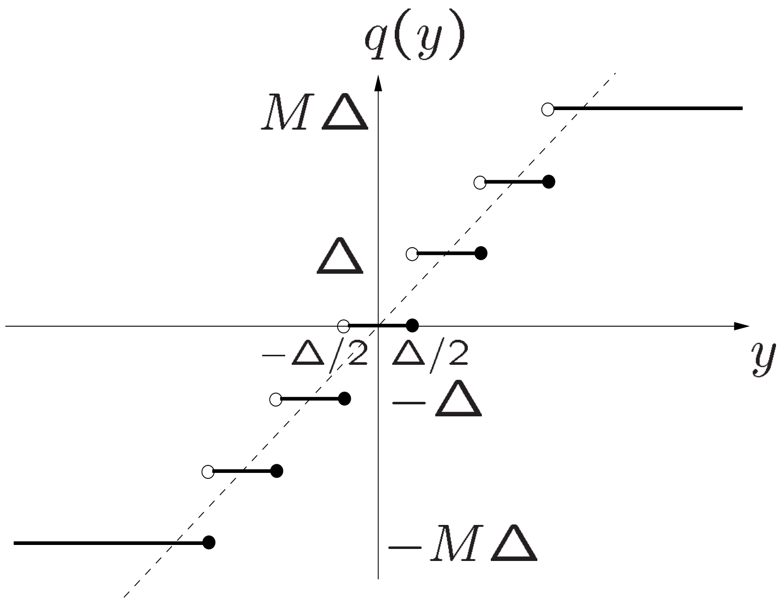
D. Delchamps (1990)

X. Feng, K. Loparo (1997)

W. Wong and coworkers

STANDARD QUANTIZER MODEL

$\mathbb{R}^1$ case:

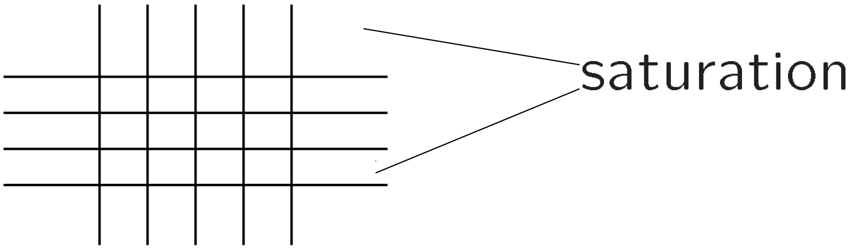


Δ – *sensitivity* of the quantizer

M – *capacity* of the quantizer

$\mathbb{R}^n$ case: $q(x) = (q_1(x_1), \dots, q_n(x_n))$.

$\mathbb{R}^n$ is partitioned into rectilinear *quantization blocks*:



We assume $\Delta_1 = \cdots = \Delta_n = \Delta$ – *uniform* quantizer.

$$\dot{x} = Ax + Bu$$

(A, B) stabilizable: $\exists K$ s.t. $A - BK$ stable

"Naive" quantized state feedback: $u = -Kq(x)$

Can't have asymptotic stability

$\exists Q, D$:

$$(A - BK)^T Q + Q(A - BK) = -D \tag{1}$$

We have

$$\dot{x} = (A - BK)x + BK(x - q(x)) \tag{2}$$

$s(x) = x - q(x)$ – the "error" vector

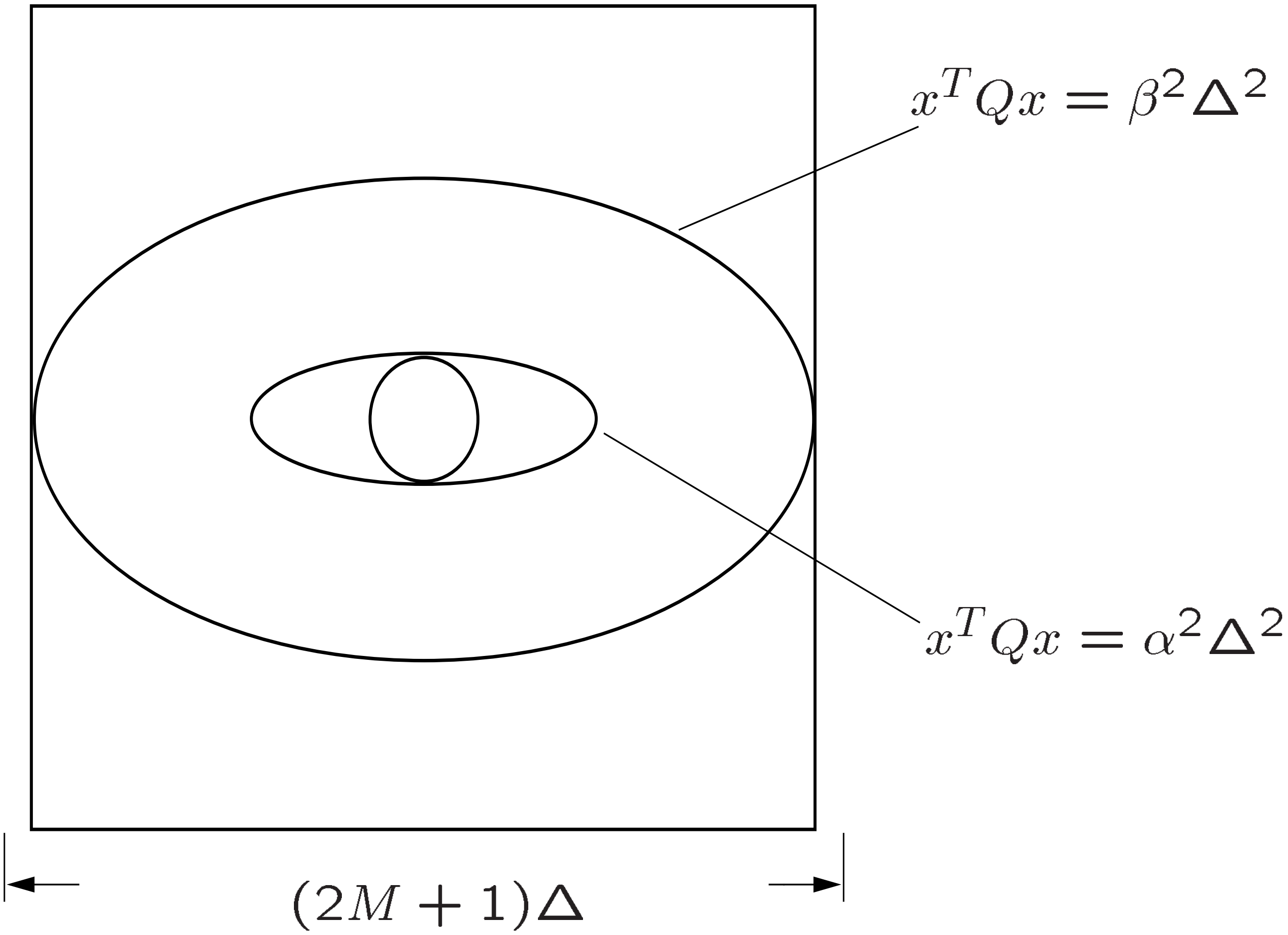
$$q \text{ doesn't saturate } \Rightarrow \|s(x)\| \le \Delta\sqrt{n}/2$$

Along solutions of quantized system (2)

$$\frac{d}{dt} x^T Q x = -x^T D x + 2x^T QBKs(x)$$

$$< 0 \quad \text{if} \quad \boxed{\|x\| > \frac{\|QBK\|}{\lambda_{min}(D)} \Delta\sqrt{n}}$$

CONTAINABILITY



M should be large enough so that $\beta > \alpha$

Delchamps

Containability (Wing Wong), practical stability

OUR QUANTIZER MODEL

Standard approach:

M fixed (often ∞)

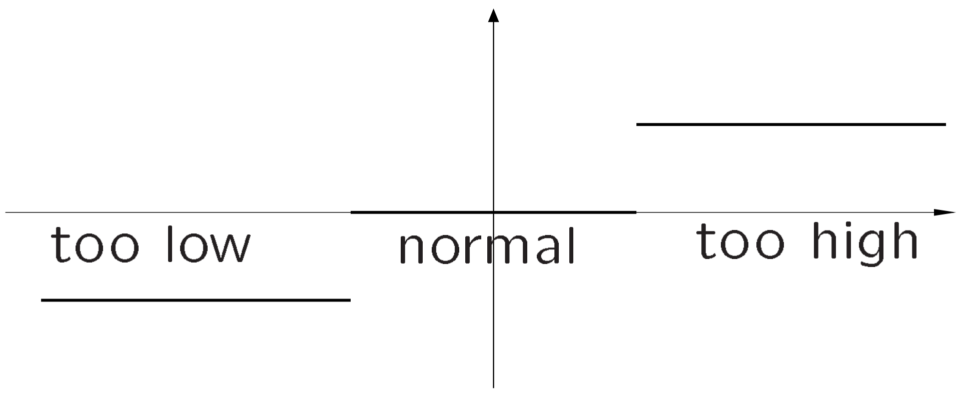
Δ fixed

Our approach:

$M < \infty$ fixed

Δ may vary while the system evolves

Example 1: temperature sensor



M is not necessarily large.

Example 2: camera with zoom

HYBRID SYSTEM APPROACH

Δ will take on a countable set of values (more realistic, robust to time delays).

The equation for Δ

$$\Delta(k\tau+\tau)=f(k,q(y(k\tau)),\Delta(k\tau)),\ \ \tau>0 \qquad (3)$$

is coupled with the given LTI system, resulting in a hybrid system.

1. "Capturing" stage – increase Δ until x belongs to the region where q doesn't saturate.

2. Control stage – decrease Δ to drive x to 0.

Example: tracking a golf ball

Goal:

stabilizable by LTI feedback

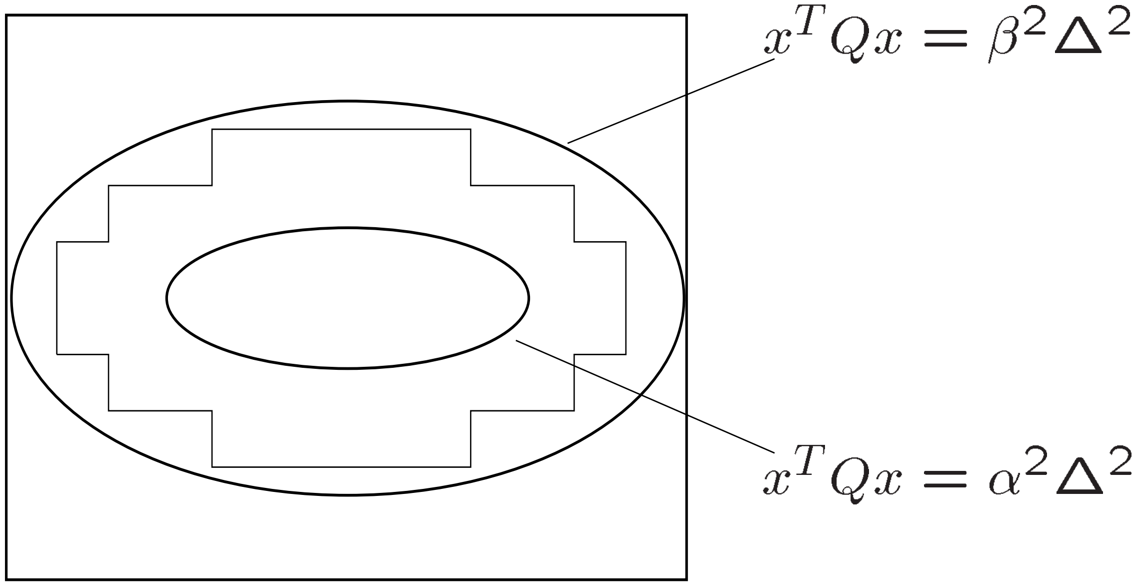
$$\Downarrow$$

stabilizable by quantized feedback

STATE FEEDBACK: M large

$$\dot{x} = Ax + Bu \qquad\qquad A - BK \text{ stable}$$

1. "Capturing" stage $(0 \le t \le t_0)$



2. Control stage $(t > t_0)$

$$u = -Kq(x)$$

$$\frac{d}{dt} x^T Q x \le -\gamma(\Delta(t_0))^2$$

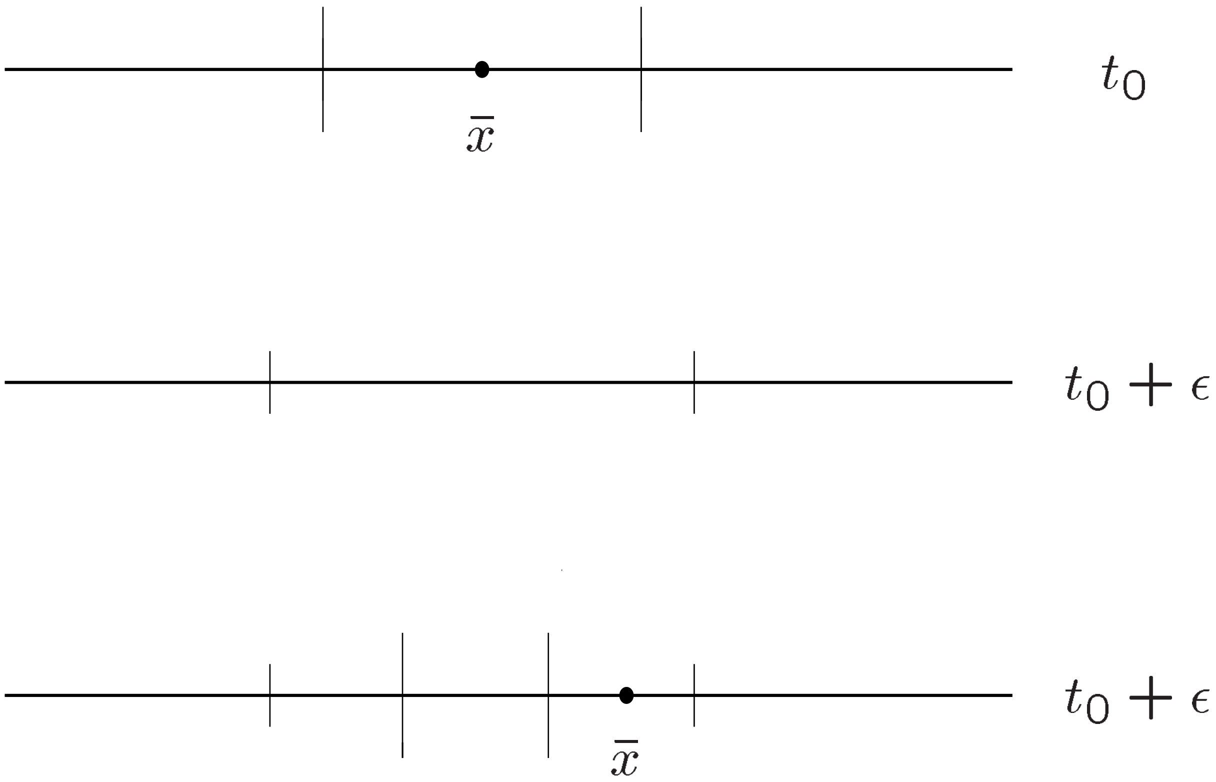
We can calculate the time t_1 such that

$$x^T(t_1) Q x(t_1) \le \alpha^2 (\Delta(t_0))^2$$

$$\text{Let} \qquad \Delta(t_1) = \frac{\alpha}{\beta} \Delta(t_0)$$

STATE FEEDBACK: M small

Have to change Δ frequently enough.



$$u(t) = -K\bar{x}(t)$$

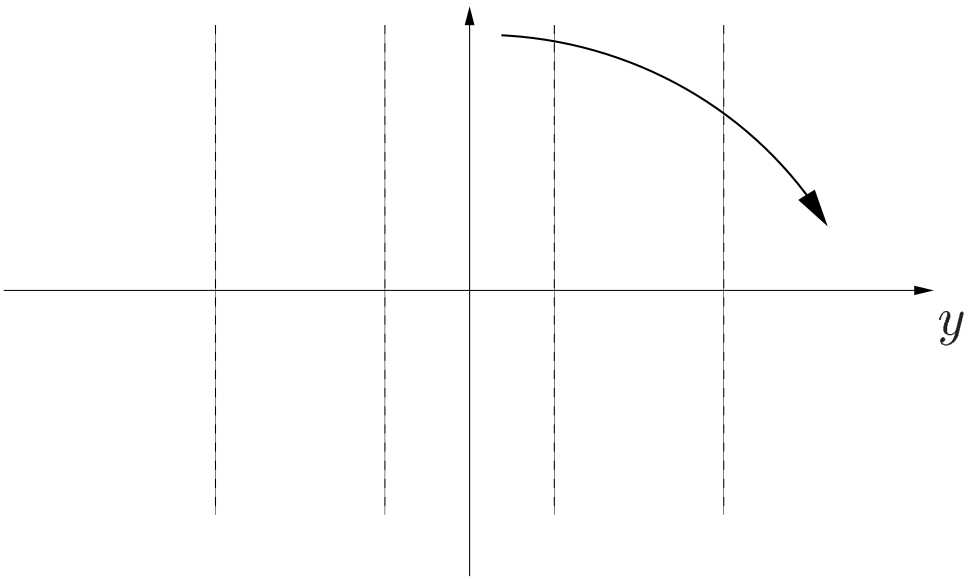
$$x - \bar{x} \to 0 \qquad \text{as } t \to +\infty$$

OUTPUT FEEDBACK

$\dot{x} = Ax + Bu$

$y = Cx$

$\exists K$ s.t. $A - BKC$ stable



Can't capture $\Rightarrow$ semiglobal stability

Assume (C, A) observable $\Rightarrow$ global stability

$$\int_{t_0}^{t_0+\tau} e^{A^T(t-t_0)} C^T y(t) dt = W x(t_0) + v \tag{4}$$

v – known, depends on $u(t)$, $t \in [t_0, t_0 + \tau]$

$W = \int_0^\tau e^{A^T t} C^T C e^{At} dt$ – observability Gramian

Define

$$\bar{x}(t_0) = W^{-1}\Big[\int_{t_0}^{t_0+\tau} e^{A^T(t-t_0)} C^T q(y(t)) dt - v\Big] \tag{5}$$

$u = -K\bar{x}$ – depends on *history* of past measurements.

I AM ALMOST DONE

Analysis of efficiency of proposed algorithms:

- time optimality
- “minimum attention” control

Extensions and generalizations:

- tracking
- computer vision
- modeling uncertainty
- discrete-time case