TOWARDS SUPERVISORY CONTROL OF UNCERTAIN NONHOLONOMIC SYSTEMS

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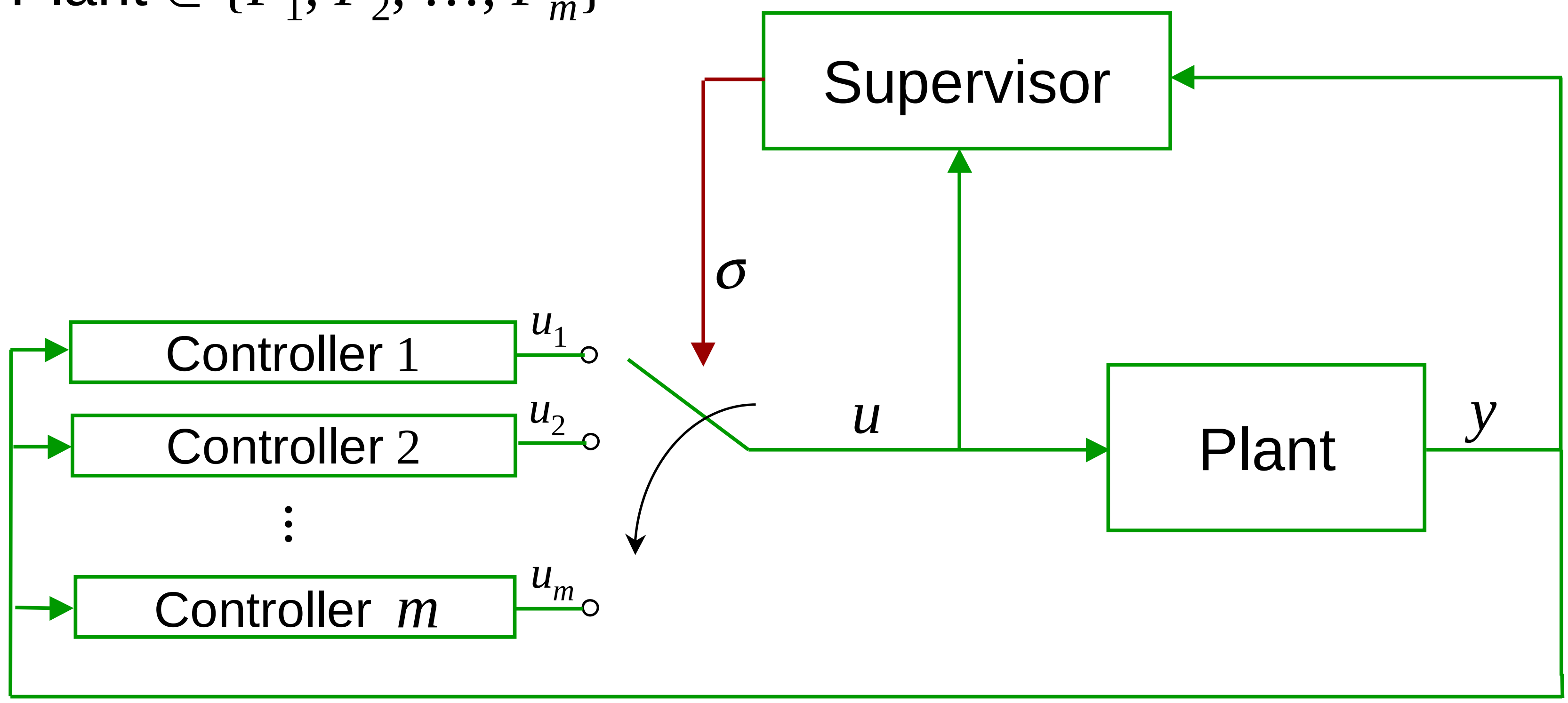
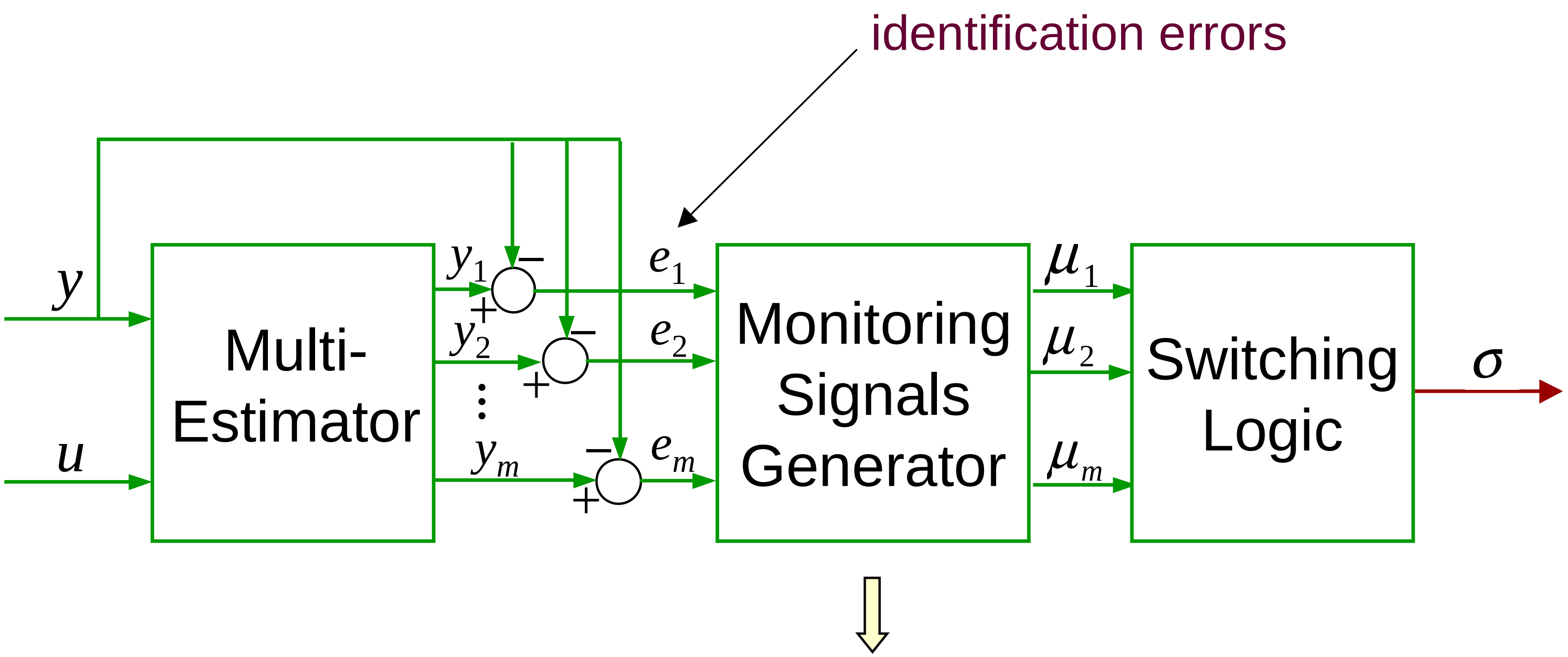
SUPERVISORY CONTROL ARCHITECTURE

Plant $\in \{P_1, P_2, \ldots, P_m\}$



σ — switching signal

SUPERVISOR



Example: weighted L^2 norm

$$\mu_i(t) = \int_0^t e^{-\lambda(t-\tau)} e_i^2(\tau)\, d\tau$$

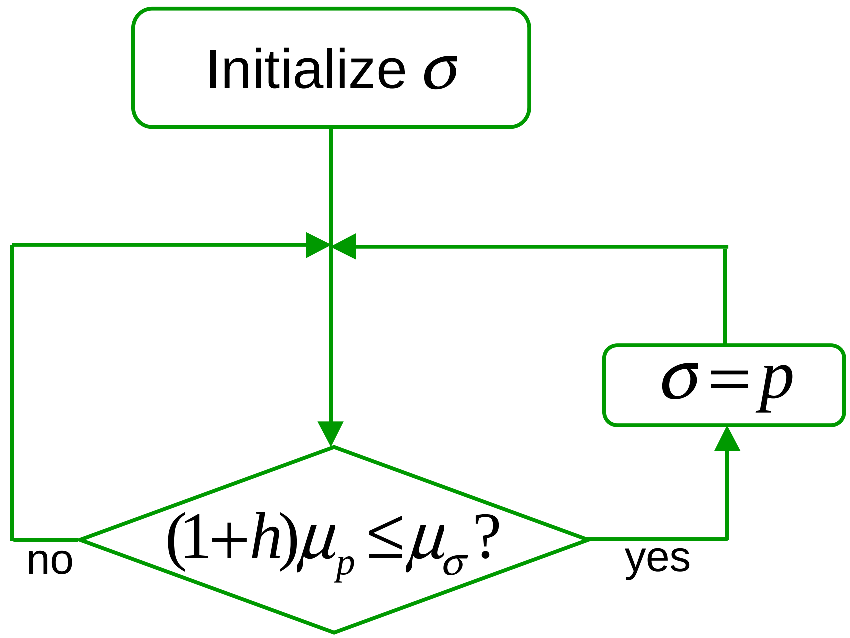
SWITCHING LOGIC

"Scale-independent hysteresis" $h > 0$ — hysteresis constant

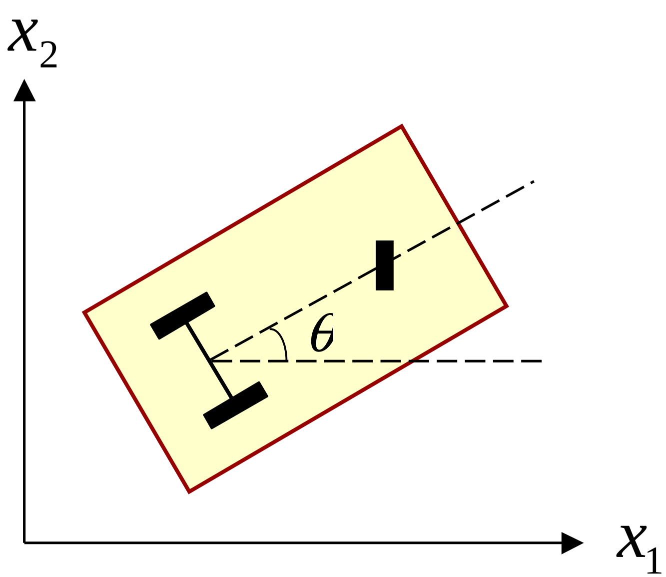
Suppose $\sigma(t_1) = q$

When $(1+h)\mu_p(t_2) \le \mu_q(t_2)$

set $\sigma(t_2) = p$



PARKING PROBLEM



$$\dot{x}_1 = p_1^* w_1 \cos\theta$$

$$\dot{x}_2 = p_1^* w_1 \sin\theta$$

$$\dot{\theta} = p_2^* w_2$$

Unknown parameters $(p_1^*, p_2^*) \in \mathcal{P}$ correspond to radius of rear wheels and distance between them

NONHOLONOMIC INTEGRATOR

Unicycle:

$$\dot{x}_1 = p_1 w_1 \cos\theta$$

$$\dot{x}_2 = p_1 w_1 \sin\theta$$

$$\dot{\theta} = p_2 w_2$$

State and control transformation

(dependent on p_1 and p_2)

Nonholonomic integrator:

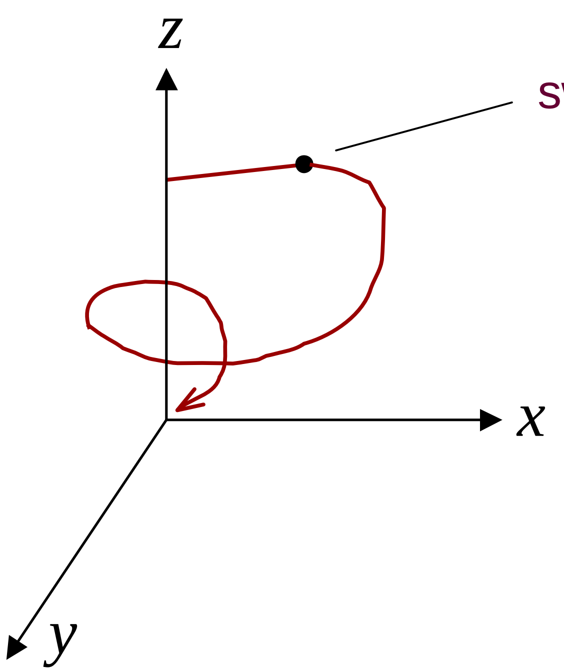
$$\dot{x} = u$$

$$\dot{y} = v$$

$$\dot{z} = xv - yu$$

HYBRID CONTROL LAWS

$$\dot{x} = u$$
$$\dot{y} = v$$
$$\dot{z} = xv - yu$$



In cylindrical coordinates:

$$\dot{r} = -r^2$$
$$\dot{\varphi} = -z / r$$
$$\dot{z} = -rz$$

MAIN RESULT

Theorem:

- All signals remain bounded;
- The switching stops in finite time;
- The state of the unicycle converges to 0.

FUTURE WORK

- Infinite parameter set:

  implementation — possible (state-sharing)

  analysis — ??? (switching might not stop)

- Noise, disturbances, unmodeled dynamics