STABILIZING A LINEAR SYSTEM WITH FINITE-STATE HYBRID OUTPUT FEEDBACK

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The Problem

Stabilizable and detectable linear system

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

Problem:

stabilize with

output feedback

(Artstein ‘96)

Switching Among Constant Gains

$$K_1, K_2, \ldots, K_m$$

Setting $u = K_i y$ we obtain

$$\dot{x} = (A + BK_i C)x$$

Let $A_i := A + BK_i C$ for $i = 1, \ldots, m$

Need switching signal σ such that

$$\dot{x} = A_\sigma x$$

is asymptotically stable

Stable Convex Combinations

$$A_1, A_2$$

Suppose:

$\alpha A_1 + (1-\alpha) A_2$ is stable for some $\alpha \in (0,1)$

Then a stabilizing switching signal exists

(Wicks, Peleties, DeCarlo)

But then the static output feedback

$$u = (\alpha K_1 + (1-\alpha) K_2) y$$

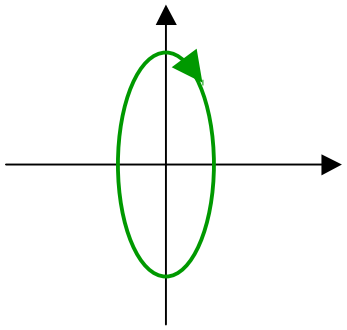
would be stabilizing

Example: Harmonic Oscillator

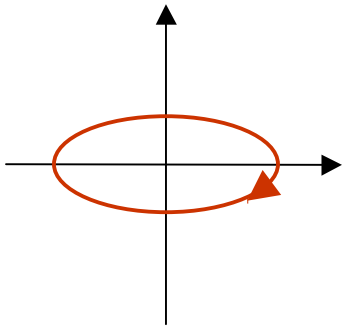
$$\ddot{y} + y = u$$

Cannot stabilize by static output feedback

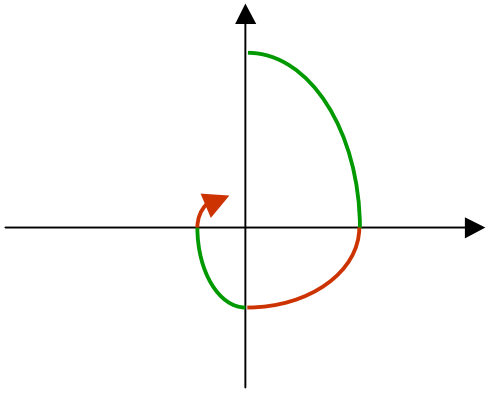
$$u = -y \quad \Rightarrow \quad \ddot{y} + 2y = 0$$



$$u = \tfrac{1}{2} y \quad \Rightarrow \quad \ddot{y} + \tfrac{1}{2} y = 0$$



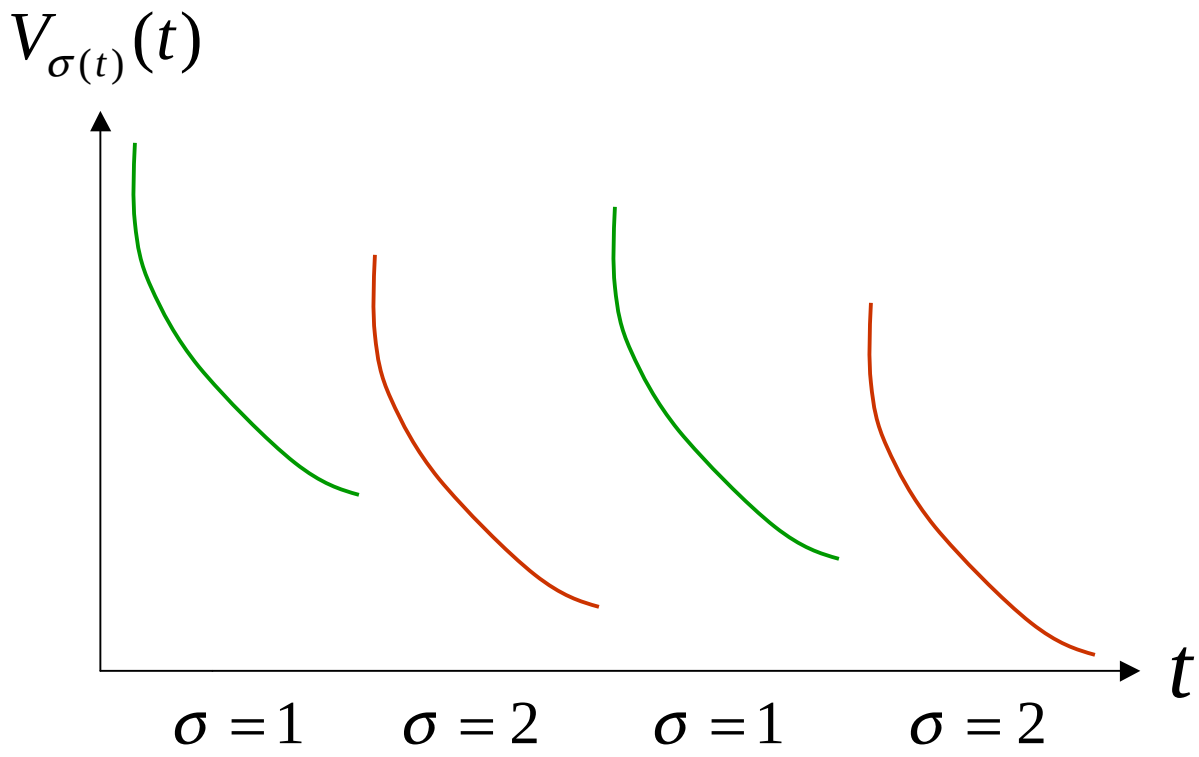
switched



Multiple Lyapunov Functions

A_1, A_2 — given matrices

V_1, V_2 — Lyapunov-like functions



Minimum Energy Switching

$$V_1 = x^T P_1 x, \quad V_2 = x^T P_2 x$$

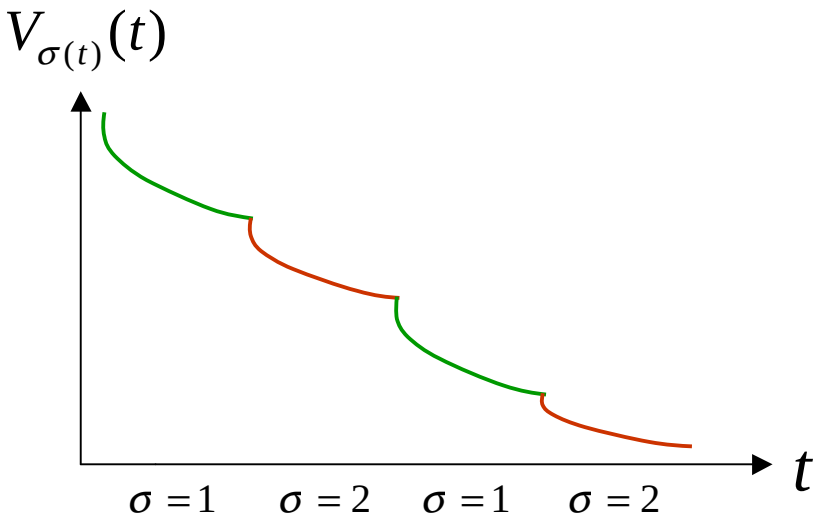
Suppose that for all $x \neq 0$ we have:

$x^T (P_1 A_1 + A_1^T P_1) x < 0$ when $x^T P_1 x \leq x^T P_2 x$

and

$x^T (P_2 A_2 + A_2^T P_2) x < 0$ when $x^T P_2 x \leq x^T P_1 x$

Then can define $\sigma(t) := \arg\min\{V_i(t) : i = 1,2\}$



Matrix Inequalities

S-procedure:

$$-P_1A_1 - A_1^TP_1 - \gamma_1(P_2 - P_1) > 0,$$
$$-P_2A_2 - A_2^TP_2 - \gamma_2(P_1 - P_2) > 0$$

Recall: $A_i = A + BK_iC$

Matrix variable elimination:

$$-P_1A_1 - A_1^TP_1 - \gamma_1(P_2 - P_1) - \delta_1P_1BB^TP_1^T > 0,$$
$$-P_1A_1 - A_1^TP_1 - \gamma_1(P_2 - P_1) - \delta_1C^TC > 0,$$
$$-P_2A_2 - A_2^TP_2 - \gamma_2(P_1 - P_2) - \delta_2P_2BB^TP_2^T > 0,$$
$$-P_2A_2 - A_2^TP_2 - \gamma_2(P_1 - P_2) - \delta_2C^TC > 0$$

Conclusion

The problem of stabilizing a linear system with finite-state hybrid output feedback is still unsolved!

Survey on switched systems:

D. Liberzon and A. S. Morse, Basic problems in stability and design of switched systems, *IEEE Control Systems Magazine*, to appear.

http://pantheon.yale.edu/~dml33/survey.ps