Stability Analysis and Stabilization of Randomly Switched Systems

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Abstract

We present sufficient conditions for almost sure global asymptotic stability (GAS a.s.) of randomly switched systems via multiple Lyapunov-like functions. For systems possessing control inputs we present a method for designing controllers which render the closed-loop randomly switched system GAS a.s.

1. The Analysis Problem

System:

$$\boxed{\dot{x} = f_\sigma(x), \qquad (x(0), \sigma(0)) = (x_0, \sigma_0), \quad t \geqslant 0} \qquad (\star)$$

- $x \in \mathbb{R}^n$, f_p vector field on $\mathbb{R}^n$, $f_p(0) = 0$, $p \in \mathcal{P}$—finite index set
- switching signal σ is a $\mathcal{P}$-valued random process

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Global asymptotic stability almost surely (GAS a.s.):
System $(\star)$ is GAS a.s. if

- $\forall \varepsilon > 0 \;\; \exists \delta > 0 : \quad |x_0| < \delta \Rightarrow \mathsf{P}\big(|x(t)| < \varepsilon \;\; \forall t \geqslant 0\big) = 1$
- $\forall r, \varepsilon' > 0 \;\; \exists T > 0 : \quad |x_0| < r \Rightarrow \mathsf{P}\big(|x(t)| < \varepsilon' \;\; \forall t \geqslant T\big) = 1$

Aim: Find sufficient conditions for GAS a.s. of $(\star)$

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Approach:

- extract properties of individual modes (via Lyapunov-like functions)
- extract statistical properties of switching signal σ
- connect the two sets of properties

2. Analysis Results: all modes stable

System: $\dot{x} = f_\sigma(x)$ $(\star)$

Theorem A.[To appear: IEEE TAC] *System* $(\star)$ *is* GAS *a.s. if*

(G1) $\exists\, C^1$ *pos def rad unbdd* $V_p : \mathbb{R}^n \longrightarrow \mathbb{R}_{\geqslant 0}$, $\exists\, \mu > 1, \lambda_\circ > 0$:

(V1) $\mathrm{L}_{f_p} V_p(x) \leqslant -\lambda_\circ V_p(x)$

(V2) $V_{p_1}(x) \leqslant \mu V_{p_2}(x)$

$N_\sigma(t) =$ # switches on $[0, t[$

(G2) $\exists\, \overline{\lambda}, \widetilde{\lambda} > 0 :\quad \mathsf{P}\big(N_\sigma(t) = k\big) \leqslant \mathrm{e}^{-\widetilde{\lambda} t}\big(\overline{\lambda} t\big)^k / k! \quad \forall\, t,\ \forall\, k$

(G3) $\mu < \big(\widetilde{\lambda} + \lambda_\circ\big)/\overline{\lambda}$

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Remarks:

- $\lambda_\circ > 0$ (no loss of generality) $\Leftrightarrow$ every mode stable
- μ standard in deterministic results, but restrictive
- (G2) is loose description—no transition probabilities involved
- (G2) $\Rightarrow$ statistically slow switching
- Continuous-time (π°, Q)–Markov chains satisfy (G2) with

$$\overline{\lambda} := \max_{i \in \mathcal{P}} |q_{ii}|, \quad \widetilde{\lambda} := \max_{i,j \in \mathcal{P}} q_{ij}$$

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Theorem B. *System* $(\star)$ *is* GAS *a.s. if*

(H1) $\exists\, C^1$ *pos def rad unbdd* $V_p : \mathbb{R}^n \to \mathbb{R}_{\geqslant 0}$, $\exists\, \mu > 1, \lambda_p \in \mathbb{R}$:

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(H2) σ *satisfies (for example):*

(S1) $(S_i)_{i\in\mathbb{N}}$, $S_i := \tau_i - \tau_{i-1}$, *is an i.i.d exponential*(λ)[1] *sequence*

(S2) $(\sigma(\tau_i))_{i\in\mathbb{N}}$ *is an i.i.d sequence, with* $\mathsf{P}\big(\sigma(\tau_1) = p\big) = q_p$, $p \in \mathcal{P}$

(S3) $(S_i)_{i\in\mathbb{N}}$ *is independent of* $(\sigma(\tau_i))_{i\in\mathbb{N}}$

(H3) $\lambda + \lambda_p > 0$ *and* $\sum_{p\in\mathcal{P}} \big(\mu q_p / \big(1 + \lambda_p/\lambda\big)\big) < 1$

[1] Density function $f_{S_i}(s) = \lambda \mathrm{e}^{-\lambda s}$ if $s \geqslant 0$, and 0 else

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Note:
- λ fixed $\Rightarrow \lambda_p > -\lambda \;\; \forall\, p$ (maximal allowable instability)
- $\lambda_p \to -\lambda$ (greater instability) $\Rightarrow q_p \to 0$

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- (H1) $\Rightarrow V_{\sigma(\tau_\nu)}(x(\tau_\nu)) \leqslant \mu V_{\sigma(\tau_{\nu-1})}(x(\tau_{\nu-1}))\mathrm{e}^{-\lambda_{\sigma(\tau_{\nu-1})} S_\nu}$ a.s.

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- Take expectations

$$\mathsf{E}\big[V_{\sigma(\tau_\nu)}(x(\tau_\nu))\big] \leqslant V_{\sigma_0}(x_0)\mathsf{E}\left[\prod_{i=0}^{\nu-1} \mu \mathrm{e}^{-\lambda_{\sigma(\tau_i)} S_{i+1}}\right] = V_{\sigma_0}(x_0) \prod_{i=0}^{\nu-1} \mu \mathsf{E}\big[\mathrm{e}^{-\lambda_{\sigma(\tau_i)} S_{i+1}}\big]$$

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- $\mathsf{E}\left[\mathrm{e}^{-\lambda_{\sigma(\tau_i)} S_{i+1}}\right] = \sum_{p \in \mathcal{P}} \int_0^\infty \lambda \mathrm{e}^{-(\lambda_p + \lambda) v} \mathsf{P}\big(\sigma(\tau_i) = p\big)\, \mathrm{d}v = \sum_{p \in \mathcal{P}} \big(q_p / (1 + \lambda_p/\lambda)\big)$

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- $\mathsf{E}\big[V_{\sigma(\tau_\nu)}(x(\tau_\nu))\big] \leqslant V_{\sigma_0}(x_0)\big(\sum_{p\in\mathcal{P}} \mu q_p/\big(1 + \lambda_p/\lambda\big)\big)^\nu \to 0$ as $\nu \to \infty$ by (H3)
- With more work GAS a.s. can be deduced from here

4. Synthesis Results

System:

$$\dot{x} = f_\sigma(x) + \sum_{i=1}^{m} g_{\sigma,i}(x)u_i, \qquad (x(0),\sigma(0)) = (x_0,\sigma_0), \quad t \geqslant 0 \qquad (\dagger)$$

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- Mode-independent ($u = k(x)$)
  - analysis results still usable since they allow unstable modes
  - search for u: some modes stabilized, others not too destabilized

5. Work in Progress + Future Directions

- Input-to-state disturbance attenuation under random switching
- Controller synthesis under partial information of σ
- Extending results to Markovian jump systems
- Detailed proofs in extended report available at:
  http://decision.csl.uiuc.edu/~liberzon/publications.html