An Erratum on Universal construction of feedback laws achieving ISS and integral-ISS disturbance attenuation

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Abstract

Lemma 5 in [2] was incorrect. A correction is provided here. The error does not affect the rest of the results in the paper.

1 The Error and the Fix

There is an error in the statement and the proof of Lemma 5 in the paper [2]. The estimate about $\alpha_0(|z(t)|)$ towards the end of the proof of Lemma 5 is incorrect. It should be replaced by

$$\begin{aligned} \alpha_0(|z(t)|) &\le \beta(|z_0|, \tau(t)) + \int_0^{\tau(t)} \gamma(|d_x(s)|)\, ds \\ &\le \beta(|z_0|, \tau(t)) + \int_0^{t} \gamma(|d(s)|)\varphi(z(s))\, ds. \end{aligned}$$

Without knowing whether $\varphi(|z(s)|)$ is bounded, one cannot conclude that $\alpha_0(|z(t)|)$ is bounded.

Below we provide a correction of the lemma by posing a boundedness condition on the function φ.

Lemma 1 *Consider a general system*

$$\dot{x} = f(x, d), \tag{1}$$

where f is a locally Lipschitz function and d is a locally essentially bounded disturbance input. Suppose the system is iISS. Let $\varphi : \mathbb{R}^n \to \mathbb{R}$ be a bounded, positive definite and smooth function. Then the system

$$\dot{x} = \varphi(x) f(x, d) \tag{2}$$

is iISS.

Remark 1 The change in the lemma does not affect the consequent results in [2]. This is because in Lemma 6 we indeed require that the function φ be bounded by 1. Hence, the main result in Section 8 of [2], Theorem 1, remains valid.

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2 The Proof

Suppose the system (1) is iISS. Then, for some $\alpha_0, \gamma \in \mathcal{K}_\infty$ and $\beta \in \mathcal{KL}$ it holds that

$$\alpha_0(|x(t)|) \leq \beta(|x(0)|, t) + \int_0^t \gamma(|d(s)|)ds \qquad \forall t \geq 0. \tag{3}$$

Pick an arbitrary initial state $z_0 \neq 0$ and a disturbance d such that

$$\int_0^\infty \gamma(|d(s)|)\, ds \leq b < \infty. \tag{4}$$

Denote by $z(\cdot)$ the corresponding solution of (2), defined on some maximal interval $[0, T)$. Let

$$\tau(t) := \int_0^t \varphi(z(s))\, ds, \qquad t \in [0, T). \tag{5}$$

Since $\varphi(z(t)) > 0$ for all $t \in [0, T)$, the function $\tau(\cdot)$ is strictly increasing. Note that τ satisfies the following initial value problem:

$$\dot{\tau}(t) = \varphi(z(t)), \quad \tau(0) = 0.$$

Hence, by the uniqueness property of the system (2), it holds that

$$z(t) = x(\tau(t)) \qquad \forall\, t \in [0, T)$$

where $x(\cdot)$ is the solution of (1) with the initial state z_0 and the disturbance function $d_x(t) := d \circ \tau^{-1}(t)$ defined on $[0, \lim_{t \to T^-} \tau(t))$. It then follows that for all $t \in [0, T)$ we have

$$\begin{aligned} \alpha_0(|z(t)|) &\leq \beta(|z_0|, \tau(t)) + \int_0^{\tau(t)} \gamma(|d_x(s)|)\, ds \\ &\leq \beta(|z_0|, \tau(t)) + \int_0^t \gamma(|d(s)|)\varphi(z(s))\, ds \leq \beta(|z_0|, \tau(t)) + Mb, \end{aligned} \tag{6}$$

where M is an upper bound of the function φ. Hence, $z(t)$ stays bounded on $[0, T)$, and consequently $T = \infty$. It is not hard to see now that the system (2) is 0-GAS and satisfies the so-called *bounded energy frequently bounded state* property (see [1]):

$$\int_0^\infty \gamma(|d(s)|)\, ds < \infty \;\Rightarrow\; \liminf_{t \to \infty} |z(t)| < \infty.$$

By Theorem 3 of [1], the system (2) is iISS. □

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References

[1] David Angeli, Brian Ingalls, Eduardo D. Sontag, and Yuan Wang. A characterization of integral input to state stability. In *Proc. 40th IEEE Conf. Decision and Control*, pages 881–886, 2001.

[2] Daniel Liberzon, Eduardo D. Sontag, and Yuan Wang. Universal construction of feedback laws achieving iss and integral-iss disturbance attenuation. *Systems & Control Letters*, 46:111–127, 2002.