Towards ISS-Disturbance Attenuation for Randomly Switched Systems

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Abstract

We provide preliminary results dealing with sufficient conditions for stochastic versions of ISS for randomly switched systems. Two types of switching signals are considered: the first is characterized by a statistically slow-switching condition, and the second by a class of semi-Markov processes.

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- σ possesses an average dwell-time τ_a if $N_\sigma(t, t') \leqslant N_0 + \dfrac{t - t'}{\tau_a} \quad (N_0 > 0)$

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Input to state stability (ISS) [Sontag]:

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Theorem 1 (Vu-Chatterjee-L, Automatica (2007)). *System* $(\star)$ *is ISS if*

- $\exists\, C^1$ *pos def* V_i *s.t.*
  - $\frac{\partial V_i}{\partial x}(x) f_i(x,d) \leqslant -\lambda_\circ V_i(x) + \chi(|d|) \quad (\lambda_\circ > 0, \chi \in \mathcal{K}_\infty)$
  - $V_i(x) \leqslant \mu V_j(x) \quad (\mu > 1)$
- σ *has average dwell-time* τ_a
- $\ln \mu < \frac{\lambda_\circ}{\tau_a}$

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ISS estimate:
System $(\star)$ is ISS in L_1 if

- $\exists\, \alpha, \gamma \in \mathcal{K}_\infty, \exists\, \beta \in \mathcal{KL} : \quad \forall\, t \quad \mathsf{E}[\alpha(|x(t)|)] \leqslant \beta(|x_0|\,, t) + \gamma(\|d\|_\infty)$

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Approach:

- extract statistical properties of switching signal σ
- extract properties of individual modes (via ISS-Lyapunov functions)
- connect the two sets of properties

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∘ $\mathsf{P}\big(N_\sigma(t, s) = k\big) \leqslant \mathrm{e}^{-\widetilde{\lambda}(t-s)} \frac{\big(\overline{\lambda}(t-s)\big)^k}{k!} \qquad \forall\, t \geqslant s \quad \forall\, k$

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- $\circ$ $\exists$ *pos. def. ubdd.* V *s.t.*
  - $\diamond$ $\frac{\partial V_i}{\partial x} f_i(x,d) \leqslant -\lambda_\circ V_i(x)$ *whenever* $|x| > \rho(|d|) \quad (\lambda_\circ > 0, \rho \in \mathcal{K}_\infty)$
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Note:
- s, T_i are optional times, P is conditional on $\mathfrak{F}_s$
- details in paper

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Switching signal: $\mathsf{P}\Big(N_\sigma(t,s) = k\Big) \leqslant \mathrm{e}^{-\widetilde{\lambda}(t-s)} \frac{\Big(\overline{\lambda}(t-s)\Big)^k}{k!}$ (†)

Define $B_1 := \big\{ y \in \mathbb{R}^n \mid |y| < \rho(\|d\|_\infty) \big\}$, a concentric $B_2 \supsetneqq B_1$, and let $x_0 \notin B_2$ (n.l.o.g)

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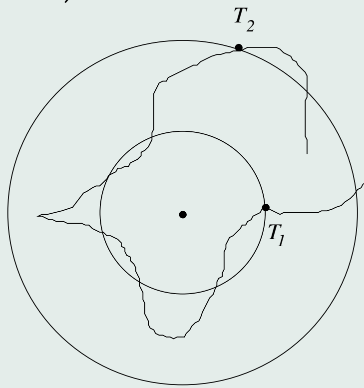

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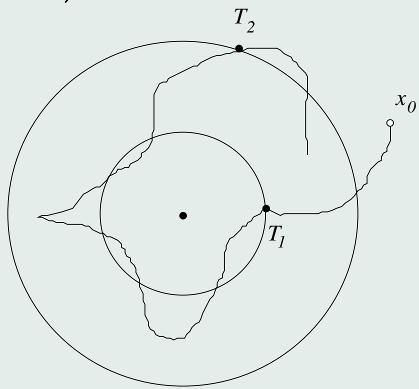

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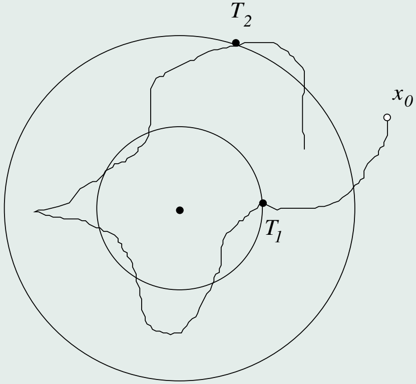

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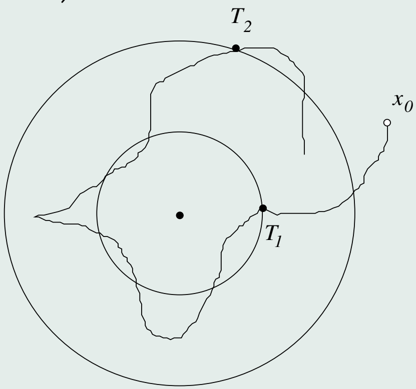

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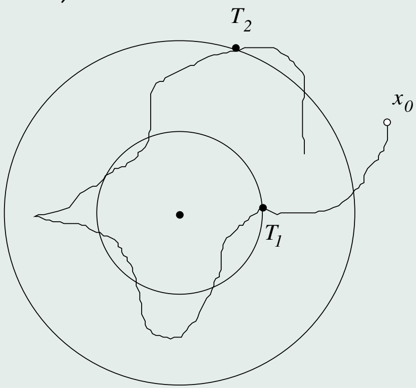

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$\because \mathrm{rad}(B_2) \sim \rho(\|d\|_\infty)$, we get the estimate from the above terms

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- σ *satisfies:*
  - *the seq. of holding times* $(S_i)_{i\in\mathbb{N}}$ *(*$S_i := \tau_i - \tau_{i-1}$*) is i.i.d,* unif$(T)$
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  - ◇ $V_i(x) \leqslant \mu V_j(x) \quad (\mu > 1)$
- ◦ $\sum_{i\in\mathcal{P}} \mu q_i \big(1 - \mathrm{e}^{-\lambda_i T}\big)/(\lambda_i T) < 1$

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  - $\diamond$ *the seq. of holding times* $(S_i)_{i\in\mathbb{N}}$ $(S_i := \tau_i - \tau_{i-1})$ *is i.i.d,* unif(T)
  - $\diamond$ *the jump destinations* $(\sigma(\tau_i))_{i\in\mathbb{N}}$ *is i.i.d, with* $\mathsf{P}\big(\sigma(\tau_i) = j\big) = q_j$, $j \in \mathcal{P}$
  - $\diamond$ $(S_i)_{i\in\mathbb{N}}$ *is independent of* $(\sigma(\tau_i))_{i\in\mathbb{N}}$
- $\circ$ $\exists$ *pos. def. rad. ubdd.* V_i *s.t.*
  - $\diamond$ $\frac{\partial V_i}{\partial x} f_i(x,d) \leqslant -\lambda_i V_i(x) + \chi(|d|) \qquad \underline{\lambda_i \in \mathbb{R}}, \chi \in \mathcal{K}_\infty$
  - $\diamond$ $V_i(x) \leqslant \mu V_j(x) \quad (\mu > 1)$
- $\circ$ $\sum_{i\in\mathcal{P}} \mu q_i \big(1 - \mathrm{e}^{-\lambda_i T}\big)/(\lambda_i T) < 1$

Then $\exists\, \beta \in \mathcal{KL}, \alpha, \gamma \in \mathcal{K}_\infty$ *s.t.*

$$\mathsf{E}[|x(\tau_i)|] \leqslant \beta(|x_0|, i) + \gamma(\|d\|_\infty) \qquad \forall\, i \in \mathbb{N}$$

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Switching signal: $(S_i)_{i\in\mathbb{N}}$ is i.i.d unif(T), $(\sigma(\tau_i))_{i\in\mathbb{N}}$ is i.i.d with $\mathsf{P}\big(\sigma(\tau_i) = j\big) = q_j$ $(j \in \mathcal{P})$, and $(S_i)_{i\in\mathbb{N}} \perp\!\!\!\perp (\sigma(\tau_i))_{i\in\mathbb{N}}$

Fix $i \in \mathbb{N}$.

- $V_{\sigma(\tau_{i+1})}(x(\tau_{i+1})) \leqslant \mu V_{\sigma(\tau_i)}(x(\tau_i))\mathrm{e}^{-\lambda_{\sigma(\tau_i)} S_{i+1}} + \frac{\chi(\|d\|_\infty)}{\lambda_{\sigma(\tau_i)}}\big(1 - \mathrm{e}^{-\lambda_{\sigma(\tau_i)} S_{i+1}}\big)$

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  - the second $\mathsf{E}[\cdot]$ leads to $\chi(\|d\|_\infty)$

7. ISS-type estimate under Markovian switching

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System: $\dot{x} = f_\sigma(x,d), \qquad (x(0),\sigma(0)) = (x_0,\sigma_0), \quad t \geqslant 0$ $\qquad (\star)$

- $x \in \mathbb{R}^n$, $d \in \mathbb{R}^k$, $i \in \mathcal{P}$—finite index set
- σ is a random process, $(\tau_i)_{i\in\mathbb{N}}$ are jump instants

Proposition 3. *Suppose that*

- ○ *σ is (π°, Q)-Markovian, i.e., it is the state of a continuous-time Markov chain, initial probability π° and generator matrix Q*
- ○ *$\exists$ pos. def. rad. ubdd. V on $\mathcal{P}\times\mathbb{R}^n$ s.t.*

$$|x| > \rho(|d|) \Rightarrow \mathcal{L}V(i,x) := \frac{\partial V}{\partial x}(i,x) f_i(x,d) + \sum_{j\in\mathcal{P}} q_{ij} V(j,x) \leqslant -\lambda_\circ V(i,x)$$

Then $\exists\, \beta \in \mathcal{KL}, \alpha, \gamma \in \mathcal{K}_\infty$ s.t.

$$\mathsf{E}[\alpha(|x(t)|)] \leqslant \beta(|x_0|, t) + \gamma(\|d\|_\infty) \qquad \forall\, t \geqslant 0$$

- Proof relies on martingale arguments, see [Chatterjee PhD Thesis, 2007]

8. ISS disturbance attenuation

System:

$$\dot{x} = f_\sigma(x, d) + \sum_{i=1}^{m} g_i(x) u_i, \qquad (x(0), \sigma(0)) = (x_0, \sigma_0), \quad t \geqslant 0$$

Objective: Choose u_i such that closed loop system satisfies an ISS estimate

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Two architectures:

(i) $\sigma(t)$ available at each t

(ii) σ is "unobserved"

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Approach:

- In case (i)
  - get u via ISS universal formulae [L-Sontag-Wang] for each subsystem
  - ensure sufficient stability margin (large $\lambda_\circ$) if σ is slow
  - ensure λ_i's satisfy hypotheses if σ is semi-Markovian
- In case (ii)
  - get u s.t. some subsystems are stabilized, others not too destabilised

9. Conclusion

- ISS in L_1 "discrete" estimates for general semi-Markovian signals

- ISS in L_1 estimates under semi-Markovian switching—open; issues:

  – gluing the estimates on random disjoint intervals to get uniform bound

  – interpolating random time-point estimates to get uniform bound