HYBRID CONTROL FRAMEWORK for SYSTEMS with QUANTIZATION

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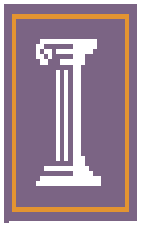

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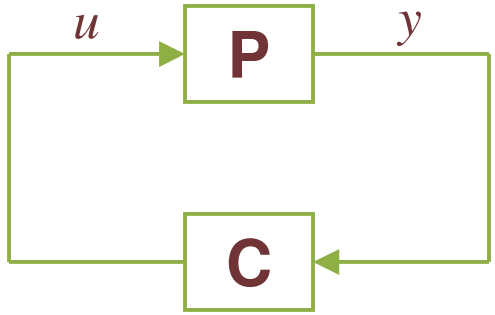
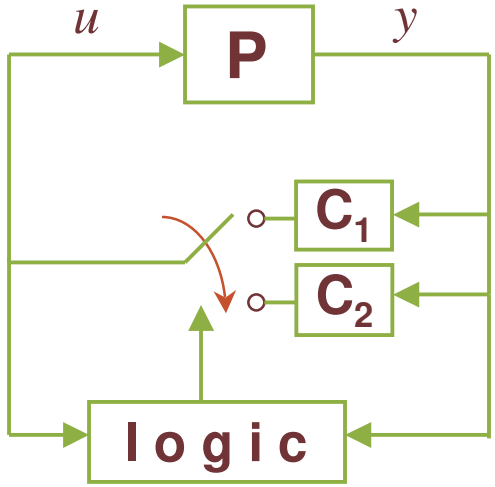
HYBRID FEEDBACK

Plant:

Classical continuous feedback paradigm:



But logical decisions are often necessary:



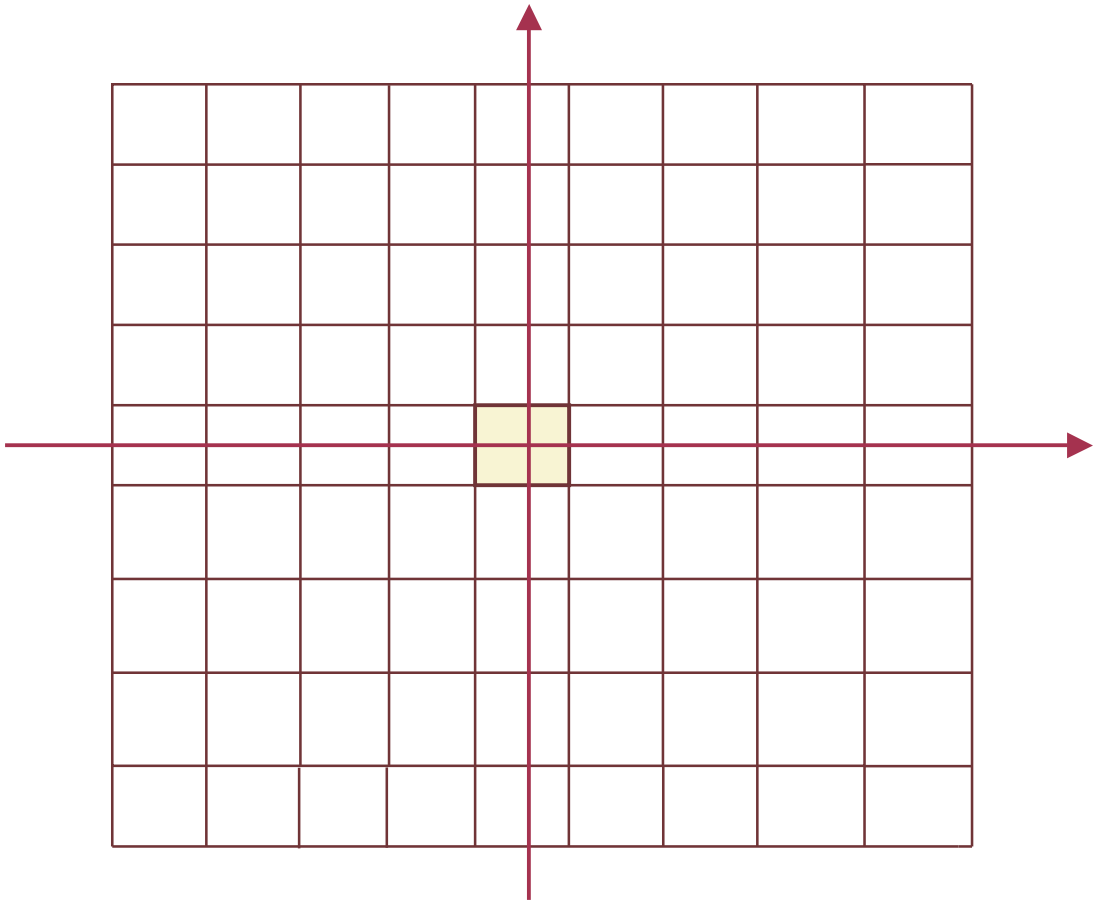
The closed-loop system is hybrid

REASONS for SWITCHING

- Nature of the control problem
- Sensor or actuator limitations
- Large modeling uncertainty
- Combinations of the above

OBSTRUCTION to STABILIZATION

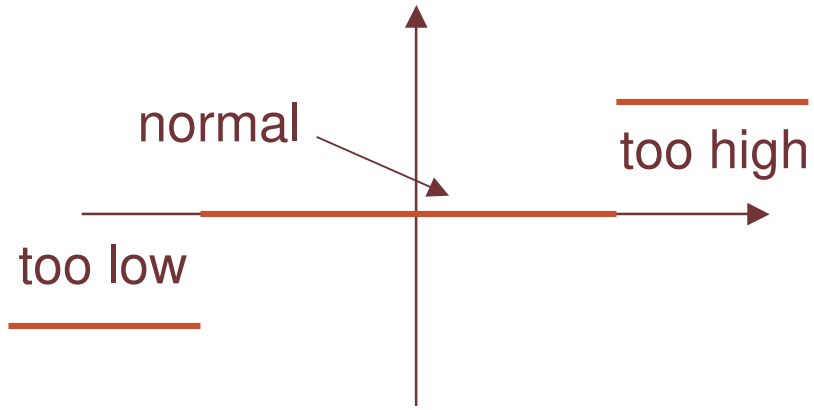
Assume:
M, Δ fixed



Asymptotic stabilization is impossible

MOTIVATING EXAMPLES

1. Temperature sensor

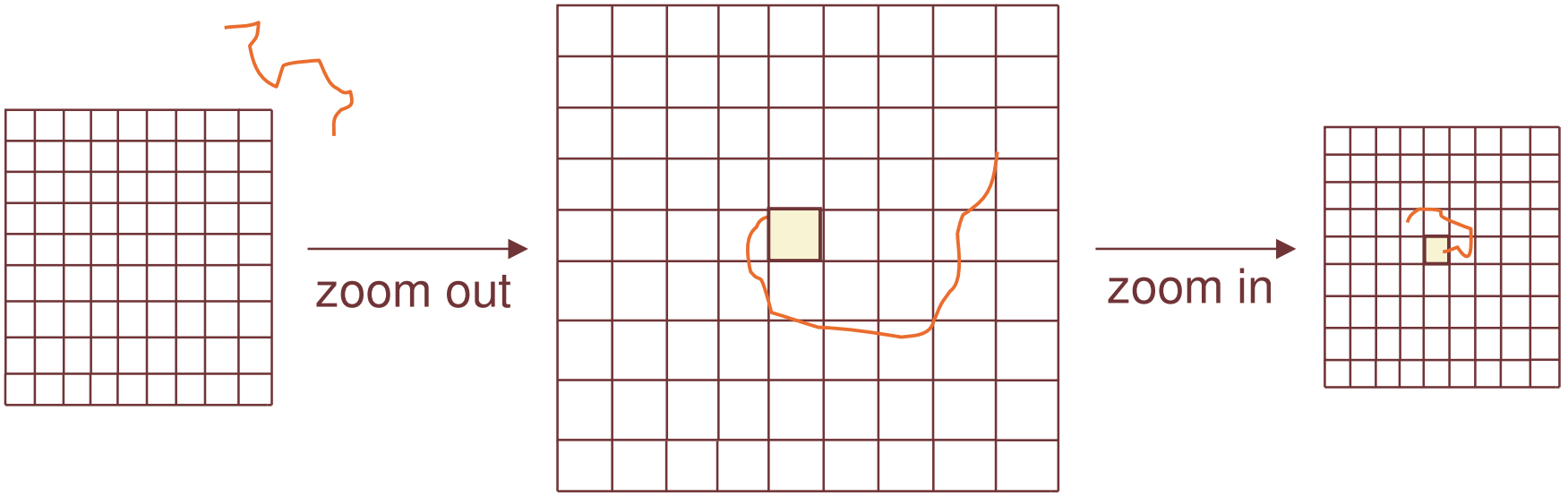


2. Camera with zoom

Tracking a golf ball

3. Coding and decoding

VARYING the SENSITIVITY



Why switch ?

- More realistic
- Easier to design and analyze
- Robust to time delays

LINEAR SYSTEMS

$$\dot{x} = Ax + Bu$$

Assume: $\exists K$ s.t. $\dot{x} = Ax + BKx$ is GAS: $\dot{V} < 0$

Along solutions of $\dot{x} = Ax + BK(x + e)$

quantization error

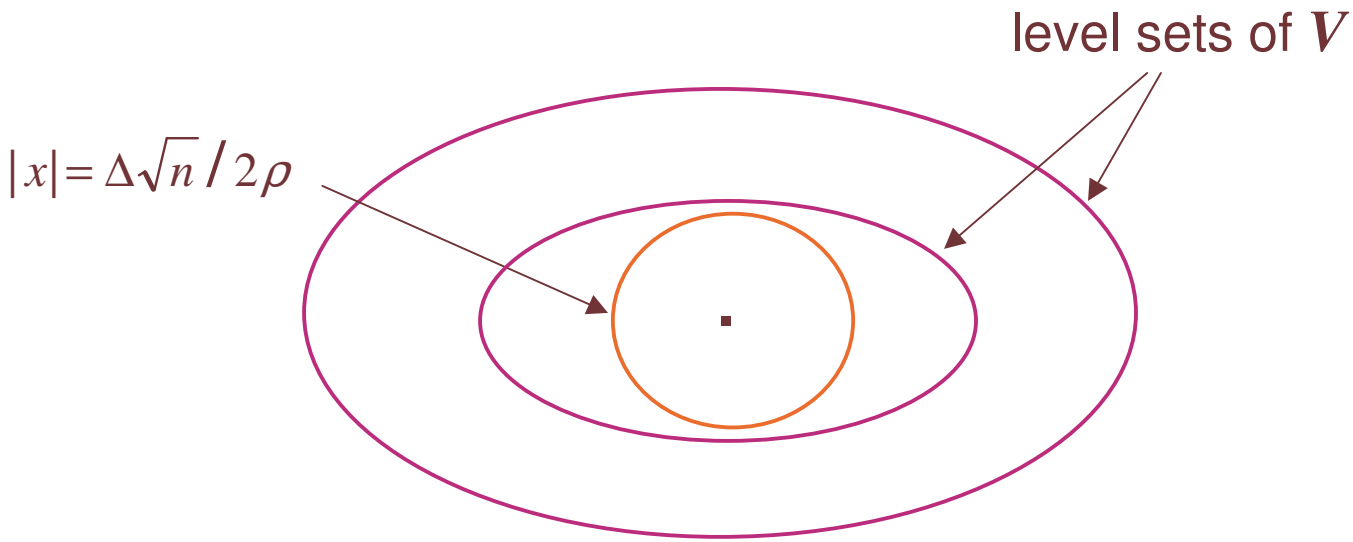
we have $|e| \le \rho |x| \Rightarrow \dot{V} < 0$ for some $\rho > 0$

Then can achieve GAS

SWITCHING POLICY

$$|e| \le \rho |x| \Rightarrow \dot{V} < 0$$

We have $|e| \le \Delta\sqrt{n}/2$



NONLINEAR SYSTEMS

$$\dot{x} = f(x,u)$$

Assume: $\exists\, k$ s.t. $\dot{x} = f(x,k(x))$ is GAS: $\dot{V} < 0$

Need: along solutions of $\dot{x} = f(x,k(x+e))$

quantization error

$$|e| \le \rho(|x|) \;\Rightarrow\; \dot{V} < 0$$

where ρ is pos. def., increasing, and unbounded

(this is input-to-state stability wrt measurement error)

Then can achieve GAS

EXTENSIONS and APPLICATIONS

- Arbitrary quantization regions
- Active probing for information
- Output and input quantization
- Relaxing the assumptions
- Application to visual servoing