DISTURBANCE ATTENUATION VIA SWITCHING

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Switched Systems

$$\boxed{\dot{x} = f_\sigma(x)}$$

compact set

where $\dot{x} = f_p(x),\ p \in \mathcal{P}$ is a family of systems

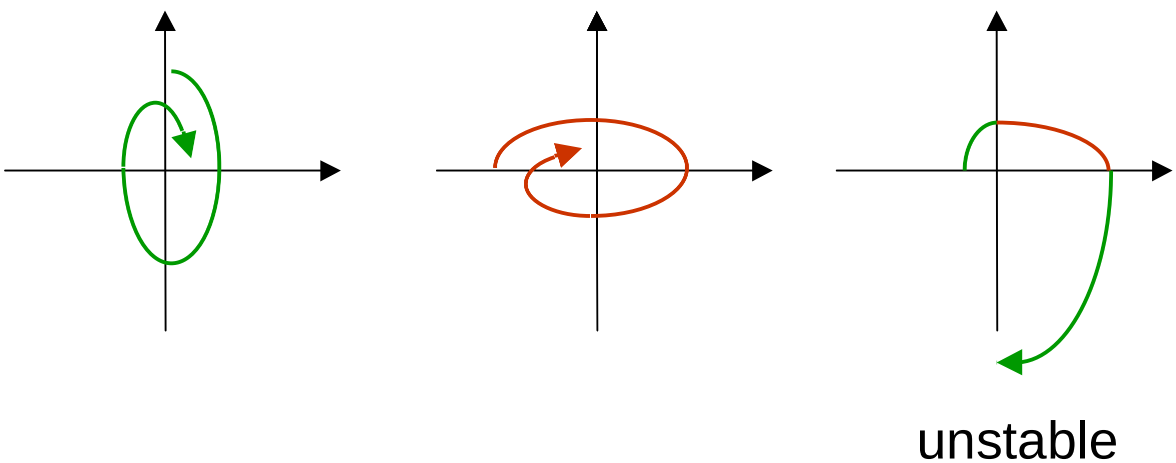
and $\sigma : [0, \infty) \to \mathcal{P}$ is a switching signal

Controller switching:

$$\dot{x} = f(x, k_\sigma(x))$$

Stability

Assume: individual subsystems are 0-GAS



A. Stability for arbitrary switching

$\exists$ common Lyapunov function

$$\Updownarrow$$

switched system is 0-GAS uniformly over σ

Stability (continued)

B. Stability for slow switching

dwell-time, average dwell-time

(Hespanha)

C. Stabilizing switching signals

single or multiple Lyapunov functions

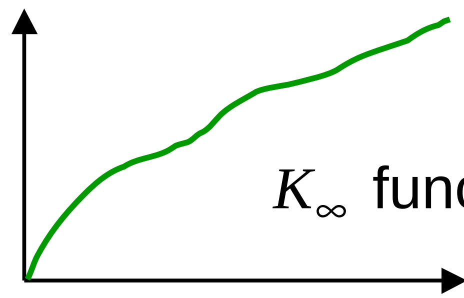
(DeCarlo et al.)

Survey on switched systems:

D. Liberzon, A. S. Morse, Basic problems in stability and design of switched systems, *IEEE Control Systems Magazine*, Oct 1999

Input-to-State Stability

$$\boxed{\dot{x} = f(x, d)}$$



ISS: for some $\beta \in KL$ and $\gamma \in K_\infty$

$$|x(t)| \leq \beta(|x(0)|, t) + \gamma\left(\sup_{0 \leq s \leq t} |d(s)|\right)$$

(Sontag '89)

Characterization: ISS-Lyap. function

$$\dot{V} \leq -\alpha(|x|) + \chi(|d|)$$

where $\alpha, \chi \in K_\infty$

Integral-Input-to-State Stability

$$\boxed{\dot{x} = f(x,d)}$$

iISS: for some $\beta \in KL$ and $\rho, \gamma \in K_\infty$

$$\rho(|x(t)|) \le \beta(|x(0)|, t) + \int_0^t \gamma(|d(s)|)ds$$

(Sontag '98)

Characterization: iISS-Lyap. function

$$\dot{V} \le -\alpha(|x|) + \chi(|d|)$$

with $\chi \in K_\infty$ and α pos def (not K_∞)

(Angeli-Sontag-Wang '98)

ISS of Switched Systems

$$\boxed{\dot{x} = f_\sigma(x, d)}$$

A. ISS for arbitrary switching

$\exists$ common ISS-Lyapunov function

$$\Updownarrow$$

switched system is ISS uniformly over σ

B. ISS for slow switching

dwell-time large enough

$$\beta(|x(0)|, t) = c\,|x(0)|\,e^{-\lambda t} \qquad \Longrightarrow \qquad \text{ISS}$$

Integral-Input-to-State Stabilization

$$\boxed{\dot{x} = f(x, d, u)}$$

C. Find u s.t. the closed-loop system is iISS

iISS-CLF: for some $\chi \in K_\infty$ and α pos def

$$\inf_u \{\nabla V(x) f(x, d, u)\} \le -\alpha(|x|) + \chi(|d|)$$

for all x, d

Want state feedback: $u = k(x)$

Switching Control

$$\boxed{\dot{x} = f(x) + g_1(x)d + g_2(x)u}$$

iISS-CLF: for some $\chi \in K_\infty$ and α pos def

$$\inf_u \{L_f V + L_{g_1} V d + L_{g_2} V u\} \le -\alpha(|x|) + \chi(|d|)$$