

is quite strange that, in this chapter, some results do concern constant delays as well. I would have preferred it considered in a separate chapter or, at least, comments and analysis could have been unified in another section.

Part III (Chapter 8) concludes the book with input–output stability analysis, mainly using the small gain theorem. First the method of comparison systems (embedding the time-delay system in a delay-free one with delay feedback) allows to get a sufficient condition of input–output stability in terms of LMIs, either using the bounded real lemma on the delay-free system with feedback uncertainty or using the LK approach using model transformation. Then a time-domain approach is used to solve a scaled small gain problem. An application of these theoretical studies concerns the approximation of time-varying or distributed delays, which are modelled as feedback uncertainty.

The interesting feature of this chapter is to make a connection with the  $H_\infty$  approach, while remaining consistent with the objective of stability analysis only.

Finally, Appendices A and B contain some useful facts on matrices and linear matrix inequalities.

In conclusion, in spite of the above criticisms, I find this book well written and very interesting to read. It contains many results on stability analysis of time-delay systems and I strongly recommend it to researchers interested in analysis and control of such systems.

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## About the reviewer

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doi :10.1016/j.automatica.2005.06.007

## Liapunov functions and stability in control theory, second ed., A. Bacciotti, L. Rosier; Springer, Berlin, 2005, ISBN: 3-540-21332-5.

Aleksandr Mikhailovich Lyapunov introduced his famous methods for investigating stability of dynamical systems more than a century ago. Basic results on Lyapunov functions are now covered in every textbook on nonlinear systems and related subjects. During the past fifty years, Lyapunov stability has been under an incessant investigation by a large community of researchers. Non-smooth Lyapunov functions, stability of systems with discontinuous/multivalued right-hand sides, behavior of systems with external inputs, and applications to feedback control are among the many issues tackled by the "modern" theory.

The purpose of this book is to present, in one self-contained and readable source, the state-of-the-art of Lyapunov theory. The book collects research results on the aforementioned topics of current interest, most of which are not typically found in introductory texts. Yet it reads closer to a textbook than to

a research monograph, which is primarily due to a large number of examples illustrating the results. The book, written by researchers who are active in this field, also contains several novel developments and examples.

The second edition retains the structure of the first one, progressing from simpler to more advanced topics and results. Chapter 1 is devoted mainly to the issue of existence of solutions for differential equations with not necessarily continuous right-hand sides. After reviewing standard existence results for Carathéodory solutions, the authors discuss Filippov's concept of generalized solution and the properties of the associated differential inclusion. Main existence results are given, with proofs.

Chapter 2 treats time-invariant systems. After reviewing the linear case, the authors discuss various stability notions for nonlinear systems and their Lyapunov characterizations. Many results and examples are given on the existence of Lyapunov functions with different degrees of regularity, not only for asymptotic stability but also for Lyapunov stability and Lagrange stability (for which converse theorems are rarely

covered in books). One thing the reviewer would have liked to see here is a proof of Lyapunov's sufficient conditions for (asymptotic) stability, since the argument is so simple yet important for the rest of the book. Other material in this chapter includes more advanced stability topics and stabilizability by feedback. This chapter has been considerably expanded compared to the first edition, and now features new topics such as LaSalle's invariance principle, Zubov's method, and a more in-depth treatment of stabilization of cascade systems.

Chapter 3 deals with time-varying systems. Obstructions to time-invariant feedback stabilization and difficulties in searching for Lyapunov functions (described in the previous chapter) provide good motivation for considering time-dependent Lyapunov functions and feedback laws. Two examples are given at the beginning of the chapter to support this point. Time-varying counterparts of stability definitions are introduced, followed by a detailed discussion of relationships between them. These relationships are explained with reference to a figure which is unfortunately not easy to understand; perhaps using a table would have been a better choice. Converse Lyapunov theorems are then presented, and results on time-varying feedback stabilization conclude the chapter.

Differential inclusions are the subject of Chapter 4. This chapter centers around the statements and proofs of converse Lyapunov results for uniform global asymptotic stability and for a strengthened version of Lyapunov stability which the authors call "robust stability". Both these theorems provide smooth Lyapunov functions. The last section of the chapter, which has been newly added in the second edition, discusses an example illustrating that a non-smooth Lyapunov function may be easier to find.

In Chapter 5, the authors return to time-invariant ordinary differential equations. Here they discuss more quantitative aspects of stability, namely, the rate of convergence (exponential convergence, rational convergence, finite-time convergence) and the question of which additive perturbations do not destroy stability. These questions are connected with the existence of Lyapunov functions that have additional properties, such as analyticity, homogeneity, or a more general symmetry. As in the rest of the book, in this chapter the reader finds many interesting examples and useful pointers to the active literature.

Several results throughout the book involve a non-differentiable Lyapunov function which must decrease along the system trajectories. Chapter 6 reviews basic tools from non-smooth analysis which can be used to check this condition. The authors define several notions of generalized

derivative and provide a helpful discussion on relationships between them. Then they explain how these notions are used to establish the decreasing condition, under various assumptions on the Lyapunov function and the right-hand side of the system.

Since basic concepts such as existence of solutions, stability of linear systems, and stability definitions for general systems are introduced in the early chapters, the reader does not need to have a lot of background in systems and control to be able to follow the book. On the other hand, it must be noted that the authors expect a certain degree of mathematical sophistication from the reader. For example, terms such as "absolutely continuous function," "set-valued map," "lower semi-continuous function" are used without being defined. Thus this book alone will probably not be sufficient for an engineering graduate student who wants to get introduced to the area. Of course, the missing background can be easily found in standard mathematical (as well as system-theoretic) texts, some of which are cited in the book. A reader with a firm grasp of basic concepts from ordinary differential equations and functional analysis will be able to follow the book and will appreciate its careful style, numerous examples, and up-to-date pointers to the literature. Thus the reviewer recommends this book as a good introduction to the subject for mathematically inclined students and researchers, as well as a unique and useful reference source for experts in the field.

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