Quantized adaptive stabilization of minimum-phase systems

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***Abstract*—We consider a linear minimum-phase system of an arbitrary relative degree with an unknown bounded disturbance and dynamically quantized measurements. The shunting method (parallel feedforward compensator) is applied to obtain hyper-minimum-phase augmented system that is further stabilized by a passification-based adaptive controller. By constructing a switching procedure for the adaptive controller parameters, we ensure convergence of the system state from an arbitrary set to an ellipsoid, whose size depends on the disturbance bound. The results are demonstrated by an example of an aircraft flight control.**

I. Introduction

If one would decide to compare two important classes of control systems: linear systems and adaptive systems, one would first come to a decision that potential of adaptive systems is higher. Indeed, adaptive systems in principle may achieve better stability and performance indices under conditions of uncertainty. However, the price for a better performance is a higher complexity. Indeed, the dynamical order of adaptive systems is typically several times higher and high order systems are more sensitive to various disturbances, unmodelled dynamics, etc. Therefore, quite a number of studies were devoted to design of simple adaptive control systems with low dynamical order of the controller [1]–[9]. The class of adaptive systems proposed in [1], [2] is based on passification: design of a feedback rendering the closed loop system passive. However, the systems of simple structure also may have limitations of their applicability. For example, for the systems based on passification, the applicability condition is passifiability: existence of a feedback rendering the closed loop system passive (in [3], [4] such a property was called (in strict version) *almost strict positive realness (ASPR)* or *almost strict passivity*). In [10] it was shown that strict passifiability is equivalent to ASPR. As it was shown in [1], a linear SIMO systems is strictly passifiable if and only if it is strictly minimum-phase and has relative degree one. In [2] this result was extended to MIMO systems based on introduction of the hyper-minimum-phaseness concept. Since the 1970s quite a number of adaptive control and synchronization problems have been solved for passifiable systems and networks (see references in [11]–[14]). Recently, an important adaptive control problem for passifiable linear systems with quantized measurements and bounded disturbances has been solved [15]. However the "relative degree one" assumption is an important limitation of the proposed solution.

In this paper we remove the "relative degree one" assumption by using the so-called shunting method (parallel feedforward compensator) in the form proposed in [16]. This allows to obtain hyper-minimum-phase augmented system, which is further stabilized by a passification-based adaptive controller. By constructing a switching procedure for the adaptive controller parameters, we ensure convergence of the system state from an arbitrary set to an ellipsoid, which size depends on the disturbance bound. The results are demonstrated by an example of an aircraft flight control.

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II. Problem statement and preliminaries

Consider an uncertain linear system

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u_p(t) + w_p(t), \\ y_p(t) &= C_p x_p(t) \end{aligned} \tag{1}$$

with the state $x_p \in \mathbb{R}^n$, control input $u_p \in \mathbb{R}$, measured output $y_p \in \mathbb{R}^l$, unknown disturbance $w_p \in \mathbb{R}^n$, and uncertain matrices A_p, B_p, C_p. The problem is to construct an adaptive controller that ensures

$$\|x_p(t)\| < \Delta_x, \quad \forall t \geq t_l \tag{2}$$

for a given Δ_x and large enough t_l.

Assumption 1: There exists $\Delta_w > 0$ such that

$$\|w_p(t)\| \leq \Delta_w, \quad \forall t \geq 0.$$

Assumption 2: There exists $g_p \in \mathbb{R}^l$ such that $g_p^T W_p(s) = g_p^T C_p(sI - A_p)^{-1} B_p$ is minimum-phase with a relative degree $r \geq 1$.

We recall that a scalar transfer function is called *minimum-phase* if its numerator is Hurwitz.

A. Passification lemma

Definition 1 ([12]): For a given vector $g \in \mathbb{R}^l$ a scalar transfer function $g^T W(s) = g^T C(sI-A)^{-1}B$ is called *hyper-minimum-phase* (HMP) if it is minimum-phase and $g^T CB > 0$.

Lemma 1 (Passification lemma [2], [11]): A rational function $g^T W(s) = g^T C(sI-A)^{-1}B$ is HMP if and only if there exist a matrix P, a vector θ_*, and a scalar $\varepsilon > 0$ such that

$$P > 0, \quad PA_* + A_*^T P < -\varepsilon P, \quad PB = C^T g, \tag{3}$$

where $A_* = A - B\theta_*^T C$.

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Remark 1: If $g^T W(s) = g^T C(sI - A)^{-1} B$ is HMP then there exists θ such that the input $u = -\theta^T y + v$ makes the system

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t)$$

strictly passive from a new input v to the output $g^T y$, i.e. there exist functions $V(x) = x^T P x$, with $P > 0$, and $\varphi(x) \geq 0$, such that $\varphi(x) > 0$ for $x \neq 0$, satisfying

$$V(x(t)) \leq V(x(0)) + \int_0^t \left[y^T(s) g v(s) - \varphi(x(s))\right] \, ds.$$

Lemma 2 ([16]): Let $g_p^T W_p(s) = g_p^T C_p(sI - A_p)^{-1} B_p$ be a minimum-phase transfer function with a relative degree $r > 1$ and a leading coefficient $g_p^T C_p A_p^{r-1} B_p > 0$. Let $P(s)$ and $Q(s)$ be Hurwitz polynomials of degrees $r-2$ and $r-1$ with positive coefficients. Then there exist a number $\kappa_0 > 0$ and a function $\lambda_0(\kappa) > 0$ such that $g_p^T W_p(s) + \kappa\lambda P(\lambda s)/Q(s)$ is HMP for any $\kappa > \kappa_0$, $0 < \lambda < \lambda_0(\kappa)$.

B. Quantizer model

Following [17] we introduce a *quantizer with a quantization range M and a quantization error bound Δ_e* as a mapping $q\colon y \mapsto q(y)$ from $\mathbb{R}^l$ to a finite subset of $\mathbb{R}^l$ such that

$$\|y\| \leq M \Rightarrow \|q(y) - y\| \leq \Delta_e. \tag{4}$$

We will refer to the quantity $e = q(y) - y$ as the *quantization error*. The concrete codomain of q is not important for our further analysis, therefore, can be chosen arbitrary. The value of M is usually dictated by the effective range of a sensor.

By *dynamic quantizer* we will mean the mapping

$$q_\mu(y) = \mu q\left(\frac{y}{\mu}\right), \tag{5}$$

where $\mu > 0$. For each positive μ one obtains a quantizer with the quantization range μM and the quantization error bound $\mu\Delta_e$. We say that M and Δ_e are the nominal quantization range and quantization error bound. We can think of μ as the "zoom" variable: increasing μ corresponds to zooming out and essentially obtaining a new quantizer with larger quantization range and quantization error bound, whereas decreasing μ corresponds to zooming in and obtaining a quantizer with a smaller quantization range but also a smaller quantization error bound.

III. Quantized adaptive control

We consider the system (1) and assume that only the quantized output $q_{\mu(t)}(y_p(t))$ is available to the controller. If there exists $g_p \in \mathbb{R}^l$ such that $g_p^T W_p(s)$ is HMP, an adaptive controller of [15] guarantees the practical stability (ultimate boundedness). However, HMP condition admits only transfer functions with relative degree $r = 1$. Below we derive an adaptive controller for minimum-phase systems with arbitrary relative degree.

Consider the system (1) subject to Assumption 2. Let us fix some Hurwitz polynomials $P(s)$ and $Q(s)$ of degrees $r-2$ and $r-1$ with positive coefficients. Due to Lemma 2, there exist λ and κ such that $g_p^T W_p(s) + W_s(s)$ with $W_s(s) = \kappa\lambda P(\lambda s)/Q(s)$ is HMP. We consider a controller that consists of a shunt system

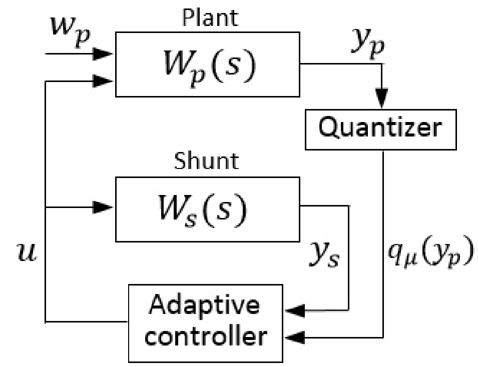

Fig. 1. A minimum-phase system with shunt

$$\begin{aligned} \dot{x}_s(t) &= A_s x_s(t) + B_s u(t), \\ y_s(t) &= C_s x_s(t) \end{aligned} \tag{6}$$

with the transfer function $W_s(s)$ and passification-based adaptive control law

$$\begin{aligned} u(t) &= -\theta_p^T(t) q_{\mu(t)}(y_p(t)) - \theta_s(t) y_s(t), \\ \dot{\theta}_p(t) &= \gamma q_{\mu(t)}(y_p(t))[g_p^T q_{\mu(t)}(y_p(t)) + y_s(t)] - a(t)\theta_p(t), \\ \dot{\theta}_s(t) &= \gamma y_s(t)[g_p^T q_{\mu(t)}(y_p(t)) + y_s(t)] - a(t)\theta_s(t), \end{aligned} \tag{7}$$

where $\theta_p \in \mathbb{R}^l$, $\theta_s \in \mathbb{R}$ are adaptive coefficients, $\gamma > 0$, and $\mu(t)$, $a(t)$ are piecewise constant (switching) parameters to be defined later.

Denote $x = \mathrm{col}\{x_p, x_s\}$, $y = \mathrm{col}\{y_p, y_s\}$, $w = \mathrm{col}\{w_p, 0\}$,

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_s \end{bmatrix}, \quad B = \begin{bmatrix} B_p \\ B_s \end{bmatrix}, \quad C = \begin{bmatrix} C_p & 0 \\ 0 & C_s \end{bmatrix} \tag{8}$$

and consider the augmented system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + w(t), \\ y(t) &= Cx(t), \end{aligned} \tag{9}$$

which transfer function is

$$W(s) = \begin{bmatrix} W_p(s) \\ W_s(s) \end{bmatrix}.$$

For $g = \mathrm{col}\{g_p, 1\}$, we obtain that $g^T W(s) = g_p^T W_p(s) + W_s(s)$ is HMP. Then Lemma 1 guarantees that there exist a matrix P, a vector $\theta_* \in \mathbb{R}^{l+1}$, and a scalar $\varepsilon > 0$ such that (3) are satisfied (with A, B, and C given in (8)). For some $\mu_0 > 0$, $V_0 > 0$ define the following quantities

$$\begin{aligned} \sigma &= \frac{\|C\|}{\mu_0 \Delta_e \|\theta_*\|} \sqrt{V_0 \lambda_{\min}^{-1}(P)}, \\ \nu &= \frac{\varepsilon}{2} - \|\theta_*\| \mu_0^2 \Delta_e^2 V_0^{-1} - 2\frac{\mu_0 \Delta_e \|\theta_*\| \|C\|}{\sqrt{\lambda_{\min}(P) V_0}}, \\ \alpha &= \varepsilon - \nu - 2\sigma^{-1} \lambda_{\min}^{-1}(P) \|C\|^2, \\ c_\gamma &= \gamma^{-1} \|\theta_*\|^2, \\ c_w &= \alpha^{-1} \nu^{-1} \lambda_{\max}(P), \\ c_e &= 2\alpha^{-1}(\|\theta_*\| + \|\theta_*\|^2 \sigma). \end{aligned} \tag{10}$$

Remark 2: By substituting σ and ν, we obtain

$$\alpha = \frac{\varepsilon}{2} + \|\theta_*\|\mu_0^2\Delta_e^2 V_0^{-1} > 0.$$

Below we will always assume that $c_e\mu_0^2\Delta_e^2 < V_0$. This is equivalent to $(\|\theta_*\| + \|\theta_*\|^2\sigma)\mu_0^2\Delta_e^2 V_0^{-1} < \alpha/2$, therefore,

$$\begin{aligned}\nu = \frac{\varepsilon}{2} - \sigma^{-1}\lambda_{\min}^{-1}(P)\|C\|^2 - (\|\theta_*\| + \|\theta_*\|^2\sigma)\mu_0^2\Delta_e^2 V_0^{-1} \\ > \frac{\varepsilon}{2} - \sigma^{-1}\lambda_{\min}^{-1}(P)\|C\|^2 - \frac{\alpha}{2} = \frac{\nu}{2}.\end{aligned}$$

The letter implies $\nu > 0$. Thus, all the quantities defined in (10) are nonnegative.

Theorem 1: Consider the system (1) subject to Assumptions 1, 2, and a dynamic quantizer with a nominal quantization range M and quantization error bound Δ_e. Let Δ_e and Δ_w be such that

$$c_w\Delta_w^2 + c_e\mu_0^2\Delta_e^2 < V_0 < \frac{\mu_0^2 M^2 \lambda_{\min}(P)}{\|C\|^2} \tag{11}$$

for some $\mu_0 > 0$ and $V_0 > 0$. For an arbitrary $\delta > 0$ let us choose some positive γ and ϵ satisfying

$$\begin{aligned}&\frac{c_\gamma + \epsilon}{1 - c_e\mu_0^2\Delta_e^2 V_0^{-1}} < \delta\lambda_{\min}(P), \\ &c_\gamma + c_w\Delta_w^2 + c_e\mu_0^2\Delta_e^2 + \epsilon < V_0\end{aligned} \tag{12}$$

and for $i \in \mathbb{N}$ define the quantities

$$\begin{aligned}&V_i = c_\gamma + c_w\Delta_w^2 + c_e\mu_{i-1}^2\Delta_e^2 + \epsilon, \\ &\mu_i = \mu_0\sqrt{V_i V_0^{-1}}, \\ &a_i = \alpha + \gamma\mu_i^2\Delta_e^2(\sigma + \|\theta_*\|^{-1}), \\ &t_i = t_{i-1} + \frac{1}{\alpha}\ln\left(1 + \frac{V_{i-1} - V_i}{\epsilon}\right),\end{aligned} \tag{13}$$

where $t_0 = 0$. Then there exists a positive integer l such that the adaptive controller (7) with the shunt (6) and the switching parameters

$$a(t) = \begin{cases} a_i, & t \in [t_i, t_{i+1}), \quad 0 \le i < l, \\ a_l, & t \ge t_l, \end{cases}$$

$$\mu(t) = \begin{cases} \mu_i, & t \in [t_i, t_{i+1}), \quad 0 \le i < l, \\ \mu_l, & t \ge t_l \end{cases}$$

guarantees

$$\|x(t)\|^2 < \frac{\lambda_{\max}(P)\Delta_w^2}{\lambda_{\min}(P)\nu^2} + \delta, \quad t \ge t_l \tag{14}$$

for the initial conditions satisfying

$$\lambda_{\max}(P)\|x(0)\|^2 + \gamma^{-1}\|\theta(0) - \theta_*\|^2 < V_0. \tag{15}$$

Moreover, $\|\theta(t)\|$ is a bounded function.

Proof: First, we show that $\{V_i\}_0^\infty$ is a decreasing sequence. Indeed, from (12), (13), we have

$$V_1 = c_\gamma + c_w\Delta_w^2 + c_e\mu_0^2\Delta_e^2 + \epsilon < V_0.$$

If $i > 0$ and for $j < i$ it has been proved that $V_j < V_{j-1}$ then

$$\begin{aligned}V_i &= c_\gamma + c_w\Delta_w^2 + c_e\frac{V_{i-1}}{V_{i-2}}\frac{V_{i-2}}{V_0}\mu_0^2\Delta_e^2 + \epsilon \\ &< c_\gamma + c_w\Delta_w^2 + c_e\frac{V_{i-2}}{V_0}\mu_0^2\Delta_e^2 + \epsilon = V_{i-1}.\end{aligned}$$

By induction, $\{V_i\}_0^\infty$ is a monotonically decreasing sequence of positive numbers and, therefore, has a limit

$$V_i \xrightarrow[i\to\infty]{} \frac{c_\gamma + c_w\Delta_w^2 + \epsilon}{1 - c_e\mu_0^2\Delta_e^2 V_0^{-1}}. \tag{16}$$

The monotonicity of V_i implies that $\{t_i\}_0^\infty$ is increasing.

Consider the augmented system (9) and the Lyapunov function

$$V(x,\theta) = x^T P x + \gamma^{-1}\|\theta - \theta_*\|^2$$

with $\theta = \mathrm{col}\{\theta_p, \theta_s\}$ and P, θ_* satisfying (3). Below we show that

$$V(t) < V_i, \quad t \ge t_i, \quad i \in \mathbb{N}_0. \tag{17}$$

The control law (7) can be written in the form

$$\begin{aligned}&u(t) = -\theta^T(t)q_{\mu(t)}(y(t)), \\ &\dot\theta(t) = \gamma q_{\mu(t)}(y(t))q_{\mu(t)}^T(y(t))g - a(t)\theta(t),\end{aligned} \tag{18}$$

where $q_\mu(y) = \mathrm{col}\{q_\mu(y_p), y_s\}$. Note that $q_\mu(y)$ formally is not a quantizer, since its codomain is not a finite set, but it satisfies the relation (4) with the same M and Δ_e as $q_\mu(y_p)$.

For $t \in [t_i, t_{i+1})$ we have

$$\begin{aligned}\dot V =& 2x^T P[Ax - B\theta^T q_{\mu_i}(y)] + 2x^T P w \\ &+ 2(\theta - \theta_*)^T q_{\mu_i}(y)q_{\mu_i}^T(y)g - 2a_i\gamma^{-1}(\theta - \theta_*)^T\theta \\ =& 2x^T P[Ax - B\theta_*^T Cx] + 2q_{\mu_i}^T(y)g(\theta_* - \theta)^T q_{\mu_i}(y) \\ &- 2e_i^T(t)g(\theta_* - \theta)^T q_{\mu_i}(y) - 2y^T g\theta_*^T e_i + 2x^T P w \\ &+ 2(\theta - \theta_*)^T q_{\mu_i}(y)q_{\mu_i}^T(y)g - 2a_i\gamma^{-1}(\theta - \theta_*)^T\theta.\end{aligned}$$

Here we used the relation $PB = C^T g$ from (3) and the notation $e_i = q_{\mu_i}(y) - y$. Let us assume that

$$|e_i(t)| \le \mu_i\Delta_e, \quad t \ge t_i, \quad i \in \mathbb{N}_0. \tag{19}$$

Without loss of generality, we assume that $\|g\| = 1$ (since for $\tilde g = g/\|g\|$ the function $\tilde g^T W(s)$ remains HMP). Then we have

$$\begin{aligned}&-2e_i^T(y)g(\theta_* - \theta)^T q_{\mu_i}(y) \\ &\le (\sigma + \|\theta_*\|^{-1})\mu_i^2\Delta_e^2\|\theta_* - \theta\|^2 + \sigma^{-1}\|y\|^2 + \|\theta_*\|\mu_i^2\Delta_e^2, \\ &-2y^T g\theta_*^T e_i \le \sigma^{-1}x^T C^T g g^T C x + \sigma\mu_i^2\Delta_e^2\|\theta_*\|^2, \\ &2x^T P w \le \nu x^T P x + \nu^{-1}\lambda_{\max}(P)\Delta_w^2, \\ &-2a_i\gamma^{-1}(\theta - \theta_*)^T\theta \le -a_i\gamma^{-1}\|\theta - \theta_*\|^2 + a_i\gamma^{-1}\|\theta_*\|^2.\end{aligned}$$

Then for $t \in [t_i, t_{i+1})$

$$\begin{aligned}\dot V + \alpha V - \beta_i \le& -(\varepsilon - \nu - 2\sigma^{-1}\lambda_{\min}^{-1}(P)\|C\|^2 - \alpha)x^T P x \\ &- (a_i - \gamma\sigma\mu_i^2\Delta_e^2 - \gamma\|\theta_*\|^{-1}\mu_i^2\Delta_e^2 - \alpha)\gamma^{-1}\|\theta_* - \theta\|^2 \\ &+ \nu^{-1}\lambda_{\max}(P)\Delta_w^2 + a_i\gamma^{-1}\|\theta_*\|^2 + \sigma\mu_i^2\Delta_e^2\|\theta_*\|^2 \\ &+ \|\theta_*\|\mu_i^2\Delta_e^2 - \beta_i.\end{aligned}$$

By taking $\beta_i = \nu^{-1}\lambda_{\max}(P)\Delta_w^2 + a_i\gamma^{-1}\|\theta_*\|^2 + (\sigma\|\theta_*\|^2 + \|\theta_*\|)\mu_i^2\Delta_e^2$ and substituting α from (10) and a_i from (13), we find that $\dot V \le -\alpha V + \beta_i$. Thus, for $t \in [t_i, t_{i+1})$,

$$V(x(t),\theta(t)) \le \left(V(x(t_i),\theta(t_i)) - \frac{\beta_i}{\alpha}\right)e^{-\alpha(t-t_i)} + \frac{\beta_i}{\alpha}. \tag{20}$$

Note that

$$\frac{\beta_i}{\alpha} = c_\gamma + c_w \Delta_w^2 + c_e \mu_i^2 \Delta_e^2 = V_{i+1} - \epsilon.$$

The relation (15) implies $V(x(0), \theta(0)) < V_0$. Thus, (17) holds for $t = t_0 = 0$. Since $\beta_0/\alpha < V_1 < V_0$, (20) implies (17) for $i = 0$. Let (17) be true for $j < i$. Then

$$V(x(t_{i-1}), \theta(t_{i-1})) < V_{i-1} \quad \text{and} \quad \frac{\beta_{i-1}}{\alpha} < V_i.$$

By using (20) on $[t_{i-1}, t_i]$, we obtain $V(x(t_i), \theta(t_i)) < V_i$. Since $\beta_i/\alpha < V_{i+1} < V_i$, (20) guarantees $V(t) < V_i$ for $t \geq t_i$. By induction, (17) holds for any $i \in \mathbb{N}_0$.

Now we show that (19) is always true. The condition (11) implies $\|y(0)\| < \mu_0 M$. Since $y(t)$ is continuous in t, if $\|y(t)\| > \mu_0 M$ for some t then the exists the smallest t_* such that $\|y(t_*)\| = \mu_0 M$. Then $\|e_0(t)\| \leq \mu_0 \Delta_e$ and, therefore, (20) implies $V(t) \leq V_0$ for $t \in [0, t_*]$. The letter implies $\|y(t_*)\| < \mu_0 M$, what contradicts to the definition of t_*. Therefore, $\|y(t)\| < \mu_0 M$ for $t > t_0$. Then (20) implies $V(x(t_1), \theta(t_1)) \leq V_1$. Since $V_1 = \mu_1^2 V_0 / \mu_0^2$, the relation (11) implies $\|y(t_1)\| < \mu_1 M$. Using the arguments similar to the above, we obtain (19) for $i = 1$. By induction, (19) holds for $i \in \mathbb{N}_0$.

The assertion of the theorem follows from (11), (12), (16). ■

Remark 3: The values of ν and σ in (10) are chosen to minimize the limit value of V_i.

Remark 4: Since the value of $\delta > 0$ in (14) is chosen arbitrary, Theorem 1 guarantees (2) for any $\Delta_x > \frac{\Delta_w}{\nu}\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$.

Remark 5: Our results are applicable to the system (1) with uncertain A_p that resides in a polytope. In this case the matrix A of the augmented system belongs to some polytope

$$A_\xi \in \left\{ \sum_{i=1}^{N} \xi_i A_i \,\middle|\, 0 \leq \xi_i, \sum_{i=1}^{N} \xi_i = 1 \right\}. \tag{21}$$

If $g^T W_\xi(s) = g^T C(sI - A_\xi)^{-1} B$ is HMP for all ξ from (21) then (3) are feasible for each ξ with some θ_ξ and P_ξ. To apply the results of this paper one should take

$$\varepsilon = \min_{\xi \in \Xi} \varepsilon_\xi, \quad \theta_* = \operatorname*{argmax}_{\theta_\xi, \xi \in \Xi} \|\theta_\xi\| \tag{22}$$

and instead of quantities $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ substitute $\min_{\xi \in \Xi} \lambda_{\min}(P_\xi)$ and $\max_{\xi \in \Xi} \lambda_{\max}(P_\xi)$, respectively. The existence of these quantities follows from Lemma 1, compactness of a set of ξ, and continuity of the matrix A_ξ in ξ.

The relations (3) are feasible for $\theta_\xi = k_* g$ with large enough k_* [12]. Since (3) are affine in A, to obtain the values from (22) one can solve linear matrix inequalities

$$P > 0,\ P(A_i - Bk_* g^T C) + (A_i - Bk_* g^T C)^T P < -\varepsilon P, \\ PB = C^T g, \quad i = 1, \ldots, N$$

with a decision variable P and tuning parameters ε, k_*.

i	t_i	V_i	$\mu_i \times 10^4$	a_i
0	0	5.2×10^4	1000	37.13
1	154.5	540.75	102	0.49
2	263.4	6.37	11	0.1
3	326.6	0.81	3.9	0.1
4	345.7	0.76	3.8	0.1

Fig. 2. Switching parameters: t_i — switching instants, V_i — upper bounds for V on $[t_i, t_{i+1})$, μ_i — zooming parameter, a_i — regularizing parameter.

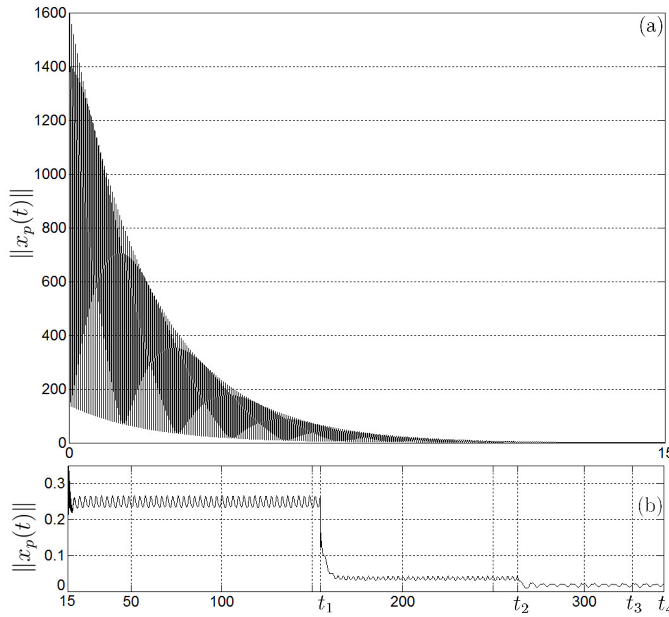

Fig. 3. Plant norm $\|x_p(t)\|$: (a) for $t \in [0, 15]$, (b) for $t \in [15, t_4]$

IV. EXAMPLE: FLIGHT CONTROL

The lateral motion of an aircraft considered as a rigid body can be described by [18]

$$\begin{aligned} \dot\beta(t) &= r(t) + a_1 \beta(t) + b_1 \delta(t), \\ \dot r(t) &= a_2 \beta(t) + a_3 r(t) + b_2 \delta(t), \\ \dot\psi(t) &= r(t), \end{aligned} \tag{23}$$

where $\psi(t)$ and $r(t)$ are the yaw angle and the yaw rate, $\beta(t)$ is the sideslip angle, $\delta(t)$ is the rudder angle (a control signal), a_i and b_i are the aircraft model parameters that depend on the flight conditions. Following [18], we take $a_3 = -1.3$, $b_1 = 19/15$, $b_2 = 19$ and assume that $a_1 \in [-1.5, -0.1]$, $a_2 \in [25, 40]$ are uncertain parameters.

The first mode of the aircraft bending is modeled as

$$W_{bend}(s) = \frac{\Delta\psi(s)}{\delta(s)} = \frac{k_{bend}}{T_{bend}^2 s^2 + 2\xi_{bend} T_{bend} s + 1}, \tag{24}$$

where $k_{bend} = -1.5 \times 10^{-3}$ is the bending mode transition factor; $T_{bend} = \omega_{bend}^{-1}$ is the response time factor with $\omega_{bend} = 65\ \mathrm{s}^{-1}$ being the first bending mode natural frequency; and $\xi_{bend} = 0.01$ is the damping ratio.

The measured signal is given by

$$y(t) = \begin{bmatrix} r(t) \\ \psi(t) + \Delta\psi(t) \end{bmatrix}. \tag{25}$$

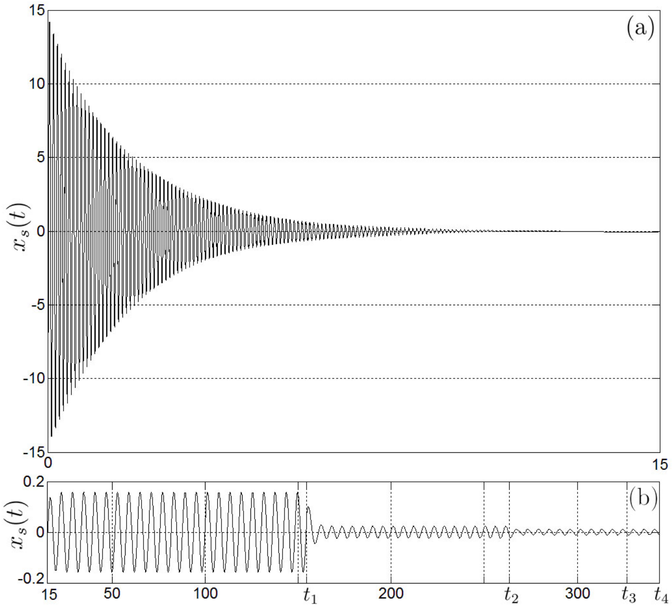

Fig. 4. Shunt state $x_s(t)$: (a) for $t \in [0, 15]$, (b) for $t \in [15, t_4]$

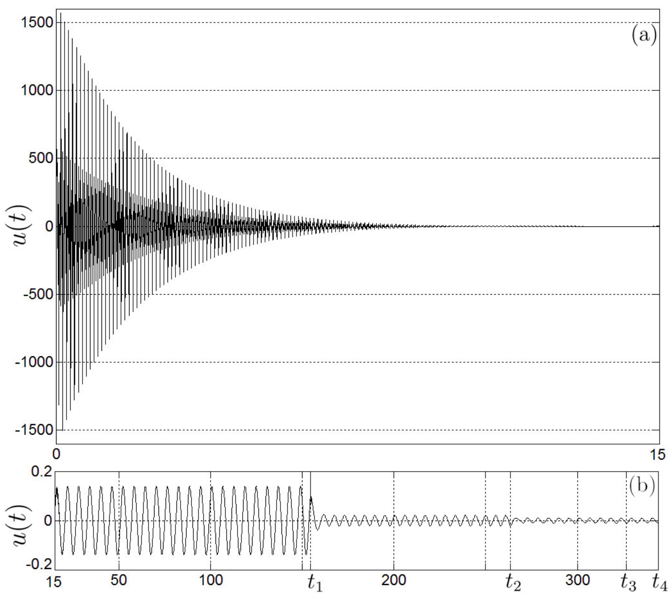

Fig. 5. Control input: (a) for $t \in [0, 15]$, (b) for $t \in [15, t_4]$

The system (23)–(25) is minimum-phase with the relative degree $r = 2$. As a shunt transfer function we take

$$W_s(s) = \frac{\kappa}{s + 1.4}$$

with $\kappa = 2$. Then for $g = \mathrm{col}\{1/2, 1/2, \sqrt{2}/2\}$ (with $\|g\| = 1$) the function $g^T W(s)$ is HMP, where $W(s)$ is the transfer function of the augmented system (9). Applying Remark 5, we find that (3) are feasible for $\varepsilon = 0.2$, $\theta_* = 27g$. Using Theorem 1 with

$$M = 10^5, \quad \Delta_e = \Delta_w = 10^{-3}, \quad \mu_0 = 0.1,$$
$$\delta = 36, \quad \epsilon = 0.01, \quad \gamma = 10^3,$$

we obtain the switching parameters presented in Fig. 2. The decrease of μ_i corresponds to "zooming in" and obtaining

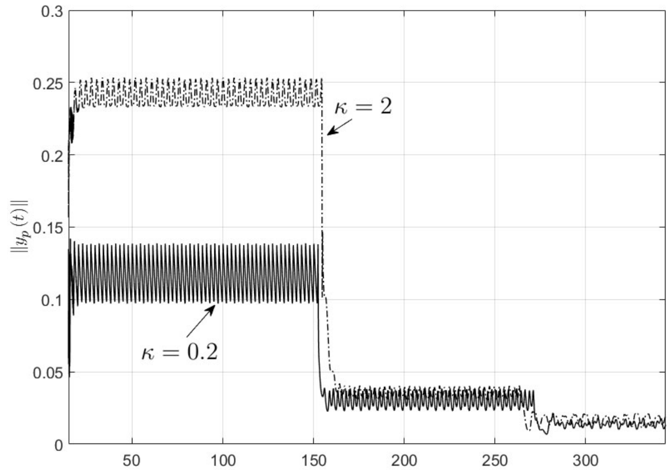

Fig. 6. The value of $\|y_p(t)\|$ on $t \in [15, 345]$ for $\kappa = 2$ and $\kappa = 0.2$

more precise measurements. The limit value for the Lyapunov function estimates is $V_\infty = 0.747$ and the limit upper bound (14) is $\|x(t)\| \leq 6$.

The results of numerical simulations for $a_1 = -0.75$, $a_2 = 33$ are presented in Fig. 3–5. Note that at switching instants $t_1, t_2, \ldots$ the dynamics of the state significantly change. This happens due to switching of the zooming parameter μ_i and regularizing parameter a_i.

In Fig. 6 one can see the norm of the plant output for different values of the shunt gain κ. The plant output is smaller for small κ, since the shunt contribution to the system dynamics gets smaller. On the other hand, $\kappa = 0.2$ requires larger gain $\theta_* = 73g$ that ensures feasibility of (3). As a result, the guaranteed limit set gets larger: $\|x(t)\| \leq 18$.

V. Conclusions

In this paper an approach to adaptive control for linear minimum-phase systems of an arbitrary relative degree with an unknown bounded disturbance and dynamically quantized measurements is proposed. Our approach is based on the shunting method (parallel feedforward compensator) that is applied to render the linear minimum-phase system of an arbitrary relative degree hyper-minimum-phase. An augmented system is further stabilized by a passification-based adaptive controller allowing one to deal with an unknown bounded disturbance and dynamically quantized measurements. By constructing a switching procedure for the adaptive controller parameters, we ensure convergence of the system state from an arbitrary set to an ellipsoid, which size depends on the disturbance bound. The simulation results demonstrate that the smaller value of κ leads to a more conservative limit set estimate.

The issue of optimization of the system parameter choice is the matter of the future research.

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