LIE-ALGEBRAIC STABILITY CRITERIA FOR SWITCHED SYSTEMS

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Switched Systems

$$\boxed{\dot{x} = f_\sigma(x)}$$

compact set

where $\dot{x} = f_p(x),\ p \in \mathcal{P}$ is a family of systems

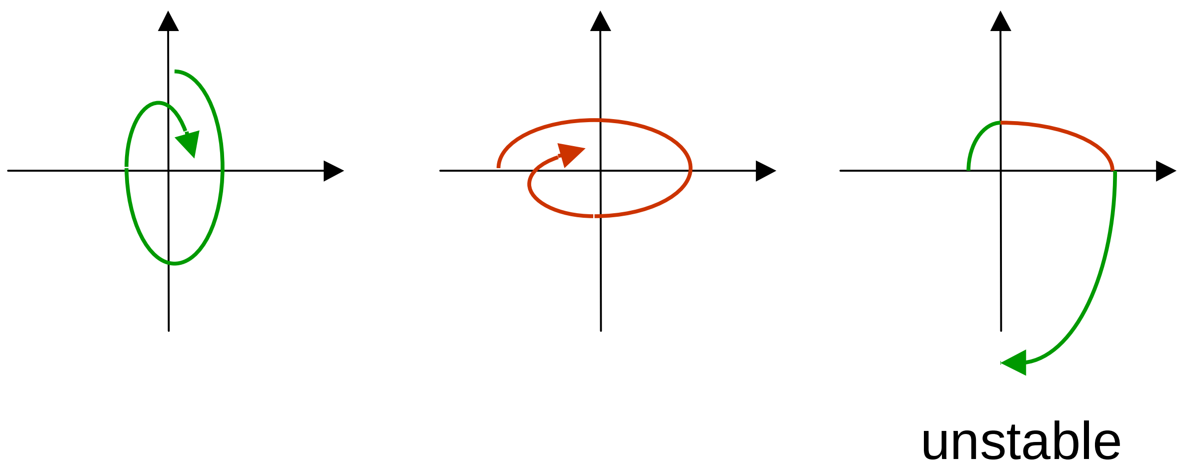
and $\sigma : [0,\infty) \to \mathcal{P}$ is a switching signal

Switched linear system:

$$\boxed{\dot{x} = A_\sigma x}$$

Uniform Exponential Stability

Assume: $A_p\ ,\ p \in \mathcal{P}$ are stable matrices



PROBLEM:

Find conditions under which we have a

GUESS: $\exists\, c, \lambda > 0$ s.t. $\forall x(0), \forall \sigma(\cdot)$

$$|x(t)| \le c e^{-\lambda t} |x(0)|$$

Commuting Matrices => GUESS

$\mathcal{P} = \{1, 2\}, \quad A_1 A_2 = A_2 A_1$

$$x(t) = e^{A_2 t_k} e^{A_1 s_k} \cdots e^{A_2 t_1} e^{A_1 s_1} x(0)$$

$$= e^{A_2 (t_k + \ldots + t_1)} e^{A_1 (s_k + \ldots + s_1)} x(0) \to 0$$

$\exists$ quadratic common Lyap fcn

(Narendra-Balakrishnan ‘94)

$$\boxed{\boldsymbol{g} := \{A_p, p \in \mathcal{P}\}_{\mathrm{LA}}}$$

Nilpotent Lie Algebra => GUESS

$\boldsymbol{g}$ is nilpotent

Gurvits '95:

$$\mathcal{P} = \{1, 2\}, \ [A_1, [A_1, A_2]] = [A_2, [A_1, A_2]] = 0$$

$$\Downarrow$$

GUESS

Baker-Campbell-Hausdorff formula

Solvable Lie Algebra => GUESS

$\boldsymbol{g}$ is solvable

Lie's Theorem: triangular form

$$A_p = \begin{pmatrix} \lambda_1 & \cdots & * \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{pmatrix}$$

$\exists$ quadratic common Lyap fcn

(L-Hespanha-Morse '98)

Solvable + Compact => GUESS

Levi decomposition: $\mathfrak{g} = \mathfrak{r} \oplus \mathfrak{s}$

radical (max solvable ideal)

THEOREM:

$\mathfrak{s}$ is compact $\Longrightarrow$ GUESS

$\exists$ quadratic common Lyap fcn

Solvable + Noncompact => ~~GUESS~~

$\boldsymbol{g} = \boldsymbol{r} \oplus \boldsymbol{s}, \quad \boldsymbol{g} \ni I, \quad \boldsymbol{s}$ is not compact

THEOREM:

- $\exists$ a set of stable generators for $\boldsymbol{g}$ that leads to a GUESS
- $\exists$ a set of stable generators for $\boldsymbol{g}$ that leads to an unstable system

Key fact: $\boldsymbol{g} \ni sl(2,\mathbb{R})$

Concluding Remarks

- Checkable conditions
- Independent of representation
- Quadratic common Lyap. fcns
- Robustness
- Nonlinear switched systems