Towards the supervisory control of uncertain nonholonomic systems*

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Abstract

This paper is concerned with control of nonholonomic systems in the presence of parametric modeling uncertainties. The specific problem considered is that of parking a wheeled mobile robot of unicycle type with unknown parameters, whose kinematics can be described by the nonholonomic integrator after an appropriate state and control coordinate transformation. We employ the techniques of supervisory control to design a hybrid feedback control law that solves this problem.

1 Introduction

Control systems with nonholonomic motion constraints have been extensively studied in the recent years, particularly in the context of robotics. Robotic manipulators, especially mobile ones, are described by complicated models about which there is likely to be significant uncertainty [4]. This is just one source of motivation for studying the challenging problem of controlling nonholonomic mechanical systems in the presence of modeling uncertainty. In the last few years this problem has received considerable attention in the literature, and several adaptive control strategies to deal with it have been proposed [3, 4, 9, 10, 15, 16].

The goal of this paper is to demonstrate that a promising alternative to conventional adaptive control for nonholonomic systems is provided by the techniques of *supervisory control* (see [6, 8] for the most recent exposition and references). The basic idea is to employ logic-based switching among a family of candidate control laws. Supervisory control makes use of standard, "off-the-shelf" controllers (for example, those obtained by feedback linearization or backstepping) rather than specialized controllers tailored to the needs of continuously tuned adaptive algorithms. This leads us to believe that various control strategies that have been developed for nonholonomic systems can be put to use in the presence of modeling uncertainties.

In what follows, we consider a prototype example of an uncertain nonholonomic control system, whose kinematics can be described by the nonholonomic integrator after an appropriate state and control coordinate transformation. We demonstrate how the supervisory control approach can be used to design a hybrid feedback control law that drives the state of the system to zero. It follows from the work described in [13] that any kinematic completely nonholonomic system with three states and two control inputs can be converted to the nonholonomic integrator by means of a state and control transformation. This result suggests that a fairly general class of nonholonomic systems can be treated by the method proposed in this paper.

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2 The problem

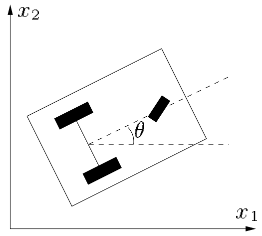

Figure 1: A wheeled mobile robot

The problem we consider is that of parking a wheeled mobile robot of unicycle type (see, e.g., [11]). Let x_1, x_2 be the coordinates of the point in the middle of the rear axle, and let θ denote the angle that the vehicle makes with the x_1-axis (see Figure 1). The front wheel rotates freely and balances the front end of the robot above the ground. When the same angular velocity is applied to both rear wheels, the robot moves straight forward. When the angular velocities applied to the rear wheels are distinct, the robot turns. The kinematics of the robot can be modeled by the equations

$$\begin{aligned} \dot{x}_1 &= p_1^* w_1 \cos\theta \\ \dot{x}_2 &= p_1^* w_1 \sin\theta \\ \dot{\theta} &= p_2^* w_2 \end{aligned} \tag{1}$$

where p_1^* and p_2^* are positive parameters determined by the radius of the rear wheels and the distance between them, and w_1 and w_2 are the control inputs. The case we are interested in is when the actual values of p_1^* and p_2^* are not precisely known. In the sequel we assume that the pair $p^* := (p_1^*, p_2^*)$ belongs to a set $\mathcal{P} := \mathcal{P}_1 \times \mathcal{P}_2$, where $\mathcal{P}_1$ and $\mathcal{P}_2$ are finite (or infinite but compact) subsets of $(0, +\infty)$. We will denote the state $(x_1, x_2, \theta)^T$ of the unicycle by $\mathbf{x}_\mathrm{u}$ and the input vector $(w_1, w_2)^T$ by $\mathbf{w}$. The problem of parking the vehicle amounts to making x_1, x_2, and θ tend to zero by means of applying a state feedback (the whole state $\mathbf{x}_\mathrm{u}$ being available for control).

Consider the state coordinate transformation

$$\begin{aligned} x &= (x_1 \cos\theta + x_2 \sin\theta) \\ y &= \theta \\ z &= 2(x_1 \sin\theta - x_2 \cos\theta) - \theta(x_1 \cos\theta + x_2 \sin\theta) \end{aligned} \tag{2}$$

It is easy to verify that (2) defines a global diffeomorphism that preserves the origin. Consider also the family of control transformations given for each $p \in \mathcal{P}$ by

$$\begin{aligned} u_p &= f_p(\mathbf{x}_\mathrm{u}, \mathbf{w}) := p_1 w_1 - p_2 w_2 (x_1 \sin\theta - x_2 \cos\theta) \\ v_p &= g_p(\mathbf{x}_\mathrm{u}, \mathbf{w}) := p_2 w_2 \end{aligned} \tag{3}$$

For $p = p^*$ the transformed equations are those of Brockett's nonholonomic integrator [2]

$$\begin{aligned} \dot{x} &= u_{p^*} \\ \dot{y} &= v_{p^*} \\ \dot{z} &= x v_{p^*} - y u_{p^*} \end{aligned} \tag{4}$$

We will denote $(x, y, z)^T$ by $\mathbf{x}_\mathrm{i}$. Following [5] (see also [14]), we further transform the state and control variables according to

$$\begin{aligned} x &= r\cos\psi \\ y &= r\sin\psi \end{aligned} \qquad \begin{pmatrix} u_p \\ v_p \end{pmatrix} = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \overline{u}_p \\ \overline{v}_p \end{pmatrix}$$

to obtain the following equations in the new cylindrical coordinates:

$$\begin{aligned} \dot{r} &= \overline{u}_{p^*} \\ \dot{\psi} &= \overline{v}_{p^*}/r \\ \dot{z} &= r\overline{v}_{p^*} \end{aligned}$$

(of course, the above transformation is only defined when $x^2 + y^2 \neq 0$). If the value of p^* were known, one could apply the controls

$$\begin{aligned} \overline{u}_{p^*} &= -r^2 \\ \overline{v}_{p^*} &= -z \end{aligned} \tag{5}$$

which would result in the system

$$\begin{aligned} \dot{r} &= -r^2 \\ \dot{z} &= -zr \\ \dot{\psi} &= -\frac{z}{r} \end{aligned} \tag{6}$$

Thus if $r(0) \neq 0$, it is not hard to see that $r(t)$, $z(t) \to 0$. If $r(0) = 0$, we could apply some control that moves the state of (4) away from the z-axis (e.g., $u_{p^*} = v_{p^*} = 1$) for a certain amount of time, and then switch to the control defined by (5). This would result in a simple hybrid control law that drives $\mathbf{x}_\mathrm{i}$, and consequently $\mathbf{x}_\mathrm{u}$, to zero. In fact, it is even possible to achieve asymptotic stability in the Lyapunov sense. Although the control law (5) is not the best one available (for example, it does not achieve exponential convergence), it provides motivation for the subsequent developments.

Since the actual parameter values p_1^* and p_2^* are unknown, the above control strategy cannot be implemented. Instead, we will develop a hybrid feedback law of the form $\mathbf{w} = \boldsymbol{\rho}_\sigma$, where $\{\boldsymbol{\rho}_p : p \in \mathcal{P}\}$ is a family of *candidate control laws*, each designed for a specific value of $p \in \mathcal{P}$, and σ is a piecewise-constant *switching signal* taking values in $\mathcal{P}$.

3 Estimator-based supervisory control

The purpose of this section is to develop a "high-level" controller called a *supervisor* which, without a priori knowledge of p^*, is capable of orchestrating the switching among a suitably defined family of candidate control laws so as to cause the state $\mathbf{x}_\mathrm{u}$ of the system (1) to tend to zero. In this section we assume that the set $\mathcal{P}$ is finite. The supervisor consists of four subsystems:

multi-estimator – a dynamical system whose inputs are $\mathbf{w}$ and $\mathbf{x}_\mathrm{i}$ and whose outputs are $\mathbf{x}_p$, $p \in \mathcal{P}$, where each $\mathbf{x}_p$ is a suitably defined estimate of $\mathbf{x}_\mathrm{i}$ which would be asymptotically correct if p were equal to p^*.

multi-controller – a dynamical system whose inputs are $\mathbf{x}_p$ and the *estimation errors* $\mathbf{e}_p := \mathbf{x}_p - \mathbf{x}_\mathrm{i}$, $p \in \mathcal{P}$, and whose outputs are the candidate control signals $\boldsymbol{\rho}_p$, $p \in \mathcal{P}$.

performance signal generator – a dynamical system whose inputs are the estimation errors $\mathbf{e}_p$ and whose outputs π_p, $p \in \mathcal{P}$ are suitably "normed" values of the estimation errors called *performance signals.*

switching logic – a dynamical system whose inputs are the performance signals π_p and whose output is a switching signal σ which is used to define the control law $\mathbf{w} = \boldsymbol{\rho}_\sigma$.

The underlying decision-making strategy used by the supervisor basically consists in selecting for σ, from time to time, the candidate control index q whose corresponding performance signal π_q is currently the smallest among the π_p, $p \in \mathcal{P}$. The motivation for this heuristic idea is as follows: the process model whose associated performance signal is the smallest "best" approximates what the process is, and thus the candidate control law designed on the basis of that model can be expected to do the best job of controlling the process.

Multi-Estimator

In view of (4), we introduce the family of estimator equations

$$\begin{aligned}\dot{x}_p &= -(x_p - x) + f_p(\mathbf{x}_\mathrm{u}, \mathbf{w})\\ \dot{y}_p &= -(y_p - y) + g_p(\mathbf{x}_\mathrm{u}, \mathbf{w})\\ \dot{z}_p &= -(z_p - z) + x g_p(\mathbf{x}_\mathrm{u}, \mathbf{w}) - y f_p(\mathbf{x}_\mathrm{u}, \mathbf{w})\end{aligned} \tag{7}$$

with x, y, z, f_p and g_p as in (2) and (3), together with the estimation errors defined by

$$\begin{aligned}\widetilde{x}_p &:= x_p - x\\ \widetilde{y}_p &:= y_p - y\\ \widetilde{z}_p &:= z_p - z\end{aligned} \tag{8}$$

For each $p \in \mathcal{P}$, let us denote $(x_p, y_p, z_p)^T$ by $\mathbf{x}_p$ and $(\widetilde{x}_p, \widetilde{y}_p, \widetilde{z}_p)^T$ by $\mathbf{e}_p$, to have $\mathbf{e}_p = \mathbf{x}_p - \mathbf{x}_\mathrm{i}$. It follows from (4) and (7) that $\dot{\mathbf{e}}_{p^*} = -\mathbf{e}_{p^*}$ for *any* control $\mathbf{w}$, thus

$$\mathbf{e}_{p^*}(t) = e^{-t}\mathbf{e}_{p^*}(0), \quad t \geq 0. \tag{9}$$

Multi-Controller

For each $p \in \mathcal{P}$, when $x_p^2 + y_p^2 \neq 0$, the equations $x_p = r_p \cos\psi_p$, $y_p = r_p \sin\psi_p$ define the signals r_p and ψ_p. There exists a function $h_p : \{(\mathbf{x}_\mathrm{u}, \mathbf{x}_p, \mathbf{e}_p) : x_p^2 + y_p^2 \neq 0\} \to \mathbb{R}^2$ such that for all $\mathbf{x}_\mathrm{u}$, $\mathbf{x}_p$ and $\mathbf{e}_p$ we have

$$\begin{aligned}f_p(\mathbf{x}_\mathrm{u}, h_p(\mathbf{x}_\mathrm{u}, \mathbf{x}_p, \mathbf{e}_p)) &= -x_p r_p + z_p \sin\psi_p + \widetilde{x}_p\\ g_p(\mathbf{x}_\mathrm{u}, h_p(\mathbf{x}_\mathrm{u}, \mathbf{x}_p, \mathbf{e}_p)) &= -y_p r_p - z_p \cos\psi_p + \widetilde{y}_p\end{aligned} \tag{10}$$

Similarly, there exists a function $\bar{h}_p : \mathbb{R}^3 \to \mathbb{R}^2$ such that for all $\mathbf{x}_\mathrm{u}$ we have

$$\begin{aligned}f_p(\mathbf{x}_\mathrm{u}, \bar{h}_p(\mathbf{x}_\mathrm{u})) &= 1\\ g_p(\mathbf{x}_\mathrm{u}, \bar{h}_p(\mathbf{x}_\mathrm{u})) &= 1\end{aligned}$$

The explicit formulas for these functions are as follows.

$$h_p(\mathbf{x}_\mathrm{u}, \mathbf{x}_p, \mathbf{e}_p) = \begin{pmatrix}\dfrac{(x_1 \sin\theta - x_2\cos\theta)\Omega + \widetilde{x}_p - x_p r_p + z_p \sin\psi_p}{p_1}\\ \Omega/p_2\end{pmatrix}$$

$$\bar{h}_p(\mathbf{x}_\mathrm{u}) = \begin{pmatrix}(1 + x_1 \sin\theta - x_2 \cos\theta)/p_1\\ 1/p_2\end{pmatrix}$$

where $\Omega = \widetilde{y}_p - y_p r_p - z_p \cos\psi_p$. Note that $\mathbf{x}_\mathrm{u}$ can be computed from $\mathbf{x}_p$ and $\mathbf{e}_p$ via (2) and (8), so that the first argument of the function h_p is actually superfluous.

Our hybrid control law will depend on two discrete state variables generated by the switching logic to be described below: $\sigma \in \mathcal{P}$ (the current estimate of p^*) and $s \in \{0, 1\}$. We define the candidate control laws $\boldsymbol{\rho}_p$, $p \in \mathcal{P}$ by

$$\boldsymbol{\rho}_p(s, \mathbf{x}_\mathrm{u}, \mathbf{x}_p, \mathbf{e}_p) := \begin{cases}h_p(\mathbf{x}_\mathrm{u}, \mathbf{x}_p, \mathbf{e}_p) & \text{if } s = 1 \text{ and } r_p \neq 0\\ \bar{h}_p(\mathbf{x}_\mathrm{u}) & \text{otherwise}\end{cases} \tag{11}$$

Loosely speaking, each of the candidate control laws $\boldsymbol{\rho}_p(s, \mathbf{x}_\mathrm{u}, \mathbf{x}_p, \mathbf{e}_p)$ makes the corresponding estimator state $\mathbf{x}_p$ reproduce the desired behavior given by (6) as closely as possible (see (15)–(16) below).

Performance signals

Define a function $F : \mathbb{R}^3 \to \mathbb{R}$ by

$$F(v) := v_1^2 + v_2^2 + v_2^6 + v_3^2.$$

Fix a positive number $\lambda < 2$. Each performance signal π_p, $p \in \mathcal{P}$ will be generated by

$$\dot{\pi}_p = -\lambda \pi_p + F(\mathbf{e}_p) \tag{12}$$

with the initial values satisfying $\pi_p(0) > 0$. The equation (12) implies that for each $p \in \mathcal{P}$ we have

$$\pi_p(t) = e^{-\lambda t}\pi_p(0) + \int_0^t e^{-\lambda(t-\tau)} F(\mathbf{e}_p(\tau)) d\tau, \qquad t \geq 0.$$

This formula shows that each performance signal π_p is the sum of an exponentially decaying term that depends on initial conditions and a suitable exponentially weighted "norm" of the corresponding estimation errors.

Switching logic

The switching logic we consider here is a modified version of the *scale-independent hysteresis switching logic* proposed in [6, 7]. Let us pick a number $h > 0$ called the *hysteresis constant*. The functioning of the switching logic is as follows (see Figure 2). First, we set $\sigma(0) = \arg\min_{p \in \mathcal{P}}\{\pi_p(0)\}$. Suppose that at a certain time t_i the value of σ has just switched to some $q \in \mathcal{P}$. If $r_q(t_i) \neq 0$, we set $s = 1$. The value of σ is then held fixed until a time $t_{i+1} > t_i$ such that $(1 + h)\min_{p \in \mathcal{P}}\{\pi_p(t_{i+1})\} \leq \pi_q(t_{i+1})$, at which point we set $\sigma(t_{i+1}) = \arg\min_{p \in \mathcal{P}}\{\pi_p(t_i + 1)\}$. (When the indicated minima are not unique, a particular value of σ among those that achieve the minimum can be chosen arbitrarily.) On the other hand, if $r_q(t_i) = 0$, we set $s = 0$ and keep the value of σ fixed for $t \in [t_i, t_i + \delta)$, where δ is a positive constant. After that we set $s = 1$ and go back to the first case. Repeating this procedure, we generate a piecewise constant signal σ that is continuous from the right everywhere. As we will see, by setting $\pi_p(0) > 0$ for all $p \in \mathcal{P}$ we avoid chattering.

Finally, using the candidate control laws (11) and the above switching logic, we define the hybrid feedback control law by

$$\mathbf{w} = \boldsymbol{\rho}_\sigma. \tag{13}$$

Our main result is as follows.

Theorem 1 *All the signals in the supervisory control system defined by (1), (7), (11), (12), (13) and the above switching logic remain bounded for any initial conditions such that $\pi_p(0) > 0\ \forall p \in \mathcal{P}$. There exists a time T^* such that $\sigma(t) = q^* \in \mathcal{P}$ for all $t \geq T^*$, i.e., the switching stops in finite time. Moreover, we have $x_1(t)$, $x_2(t)$, $\theta(t) \to 0$ as $t \to +\infty$.*

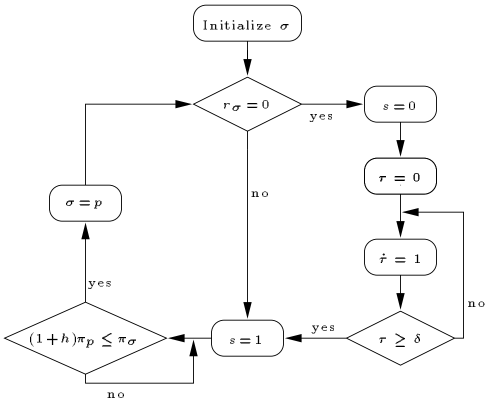

Figure 2: Switching logic

To prove this theorem, we will need the following well known and easily derived facts.

Fact 2 *The state of an exponentially stable linear system with bounded inputs, L^k inputs ($k \geq 1$) and/or inputs converging to zero remains bounded.*

Fact 3 (see, e.g., [1, p. 58]) *Any L^2 function with bounded derivative converges to zero.*

Lemma 4 *Consider the equation $\dot{x} = a(t)x + b(t)$. If $\int_0^B a(t)dt < +\infty$ and $\int_0^B |b(t)|dt < +\infty$ for all $B > 0$, then $x(t)$ remains bounded for all $t \geq 0$. Moreover, if $\lim_{B\to+\infty} \int_0^B a(t)dt = -\infty$ and $\lim_{B\to+\infty} \int_0^B |b(t)|dt = N < +\infty$, then $x(t) \to 0$ as $t \to +\infty$.*

Proof of Theorem 1. Let us define (for analysis purposes only) the scaled performance signals $\overline{\pi}_p(t) := e^{\lambda t}\pi_p(t)$, $p \in \mathcal{P}$. In view of (12) we have

$$\dot{\overline{\pi}}_p = e^{\lambda t}F(\mathbf{e}_p). \tag{14}$$

The scale independence property of the switching logic implies that replacing π_p by $\overline{\pi}_p$ would have no effect on σ. From (14) we see that each $\overline{\pi}_p$ is monotonically nondecreasing. This, the finiteness of $\mathcal{P}$, and the fact that $\overline{\pi}_p(0) > 0$ for each $p \in \mathcal{P}$ guarantee the existence of a positive number ϵ such that $\overline{\pi}_p(t) > \epsilon\ \forall t \geq 0$, $\forall p \in \mathcal{P}$. It is not hard to conclude now from the way the switching logic was defined that chattering cannot occur [6, 7].

Observe that $\overline{\pi}_{p^*}$ is bounded by virtue of (9). It follows that the signals $\overline{\pi}_p$ satisfy the assumptions of the Hysteresis Switching Lemma [6, 7, 12] which enables us to conclude that the switching stops in finite time. More precisely, if $[0, T)$ is the largest interval on which the solution to the system is defined, there exists a time $T^* < T$ such that $\sigma(t) = q^* \in \mathcal{P}$ for all $t \geq T^*$. In addition, $\overline{\pi}_{q^*}$ is bounded on $[0, T)$.

If $s(T^*) = 0$, the composite state of the system remains bounded for all $t \in [T^*, T^* + \delta]$ because the closed-loop system is globally Lipschitz. We will then have $s(T^* + \delta) = 1$. From (7), (10), (11) and (13), using straightforward calculations, we conclude that for $t \geq T^* + \delta$ we have

$$\begin{aligned} \dot{r}_{q^*} &= -r_{q^*}^2 && (15) \\ \dot{z}_{q^*} &= -z_{q^*}(r_{q^*} - \widetilde{x}_{q^*}\cos\psi_{q^*} - \widetilde{y}_{q^*}\sin\psi_{q^*}) \\ &\quad + (x_{q^*}\widetilde{y}_{q^*} - \widetilde{x}_{q^*}y_{q^*})(1 - r_{q^*}) - \widetilde{z}_{q^*} && (16) \end{aligned}$$

(here and below we are using the notation $r_p = \sqrt{x_p^2 + y_p^2}$, $p \in \mathcal{P}$). The equation (15) and the definition of the switching logic imply that $r_{q^*}(t) \neq 0$ and $s(t) = 1$ for all $t \geq T^* + \delta$. Thus we may assume, without loss of generality, that $s(T^*) = 1$ and that (15)–(16) hold for $t \geq T^*$. For each $p \in \mathcal{P} \setminus q^*$ and all $t \geq T^*$ from (7), (10), (11) and (13) we obtain the equations

$$\begin{aligned} \dot{r}_p &= -r_p + \xi_{pq^*}(\mathbf{x}_{q^*}, \mathbf{e}_{q^*}) \\ \dot{z}_p &= -z_p + \eta_{pq^*}(\mathbf{x}_{q^*}, \mathbf{e}_{q^*}) \end{aligned} \tag{17}$$

Here ξ_{pq^*} and η_{pq^*} are appropriately defined functions which satisfy the bounds

$$\begin{aligned} \xi_{pq^*}(\mathbf{x}_{q^*}, \mathbf{e}_{q^*}) &\leq P_1(r_{q^*}, \mathbf{x}_{q^*})P_2(\widetilde{x}_{q^*}) \\ &\quad + P_3(r_{q^*}, \mathbf{x}_{q^*})P_4(\widetilde{y}_{q^*}) + P_5(r_{q^*}, \mathbf{x}_{q^*})P_6(\widetilde{z}_{q^*}) \\ \eta_{pq^*}(\mathbf{x}_{q^*}, \mathbf{e}_{q^*}) &\leq P_7(r_{q^*}, \mathbf{x}_{q^*})P_8(\widetilde{x}_{q^*}) \\ &\quad + P_9(r_{q^*}, \mathbf{x}_{q^*})P_{10}(\widetilde{y}_{q^*}) + P_{11}(r_{q^*}, \mathbf{x}_{q^*})P_{12}(\widetilde{z}_{q^*}) \end{aligned} \tag{18}$$

where P_i's are suitably defined polynomials. In particular, $\deg P_2 = \deg P_6 = \deg P_8 = \deg P_{12} = 2$, $\deg P_4 = 4$, and $\deg P_{10} = 6$.

It follows from (15) that

$$r_{q^*}(t) = \frac{r_{q^*}(0)}{r_{q^*}(0)t + 1}, \qquad t \in [0, T). \tag{19}$$

Since $\overline{\pi}_{q^*}$ is bounded, we have $e^{\lambda t}F(\mathbf{e}_{q^*}) \in L^1([0, T))$. This implies, in particular, that $\widetilde{x}_{q^*}, \widetilde{z}_{q^*} \in L^2([0, T))$ and $\widetilde{y}_{q^*} \in L^2([0, T)) \cap L^6([0, T))$. Also, $\widetilde{y}_{q^*} \in L^4([0, T))$ by virtue of the bound $\widetilde{y}_{q^*}^4 \leq (\widetilde{y}_{q^*}^2 + \widetilde{y}_{q^*}^6)/2$. Moreover, from the bound $|\widetilde{x}_{q^*}| \leq (e^{-\lambda t} + e^{\lambda t}\widetilde{x}_{q^*}^2)/2$ we see that $\widetilde{x}_{q^*} \in L^1([0, T))$. Similarly, $\widetilde{y}_{q^*}, \widetilde{z}_{q^*} \in L^1([0, T))$. One can easily check now that the equation (16) satisfies the assumptions of Lemma 4. Combined with the polynomial bounds (18) and (19) this implies that $\mathbf{x}_p$ is bounded on $[0, T)$ for all $p \in \mathcal{P}$ (here we are using (17) and Fact 2). In view of (8) and (9) we conclude that $\mathbf{x}_{\mathrm{i}}$ remains bounded as well. Therefore, $\mathbf{e}_p$ is bounded for each $p \in \mathcal{P}$. The boundedness of all the performance signals now follows from Fact 2 applied to (12). Thus we see that $T = +\infty$, i.e., the solution to the system is globally defined.

Now (15)–(16) and Lemma 4 imply that $\mathbf{x}_{q^*} \to 0$. Furthermore, since $\mathbf{x}_{\mathrm{i}}$ and $\mathbf{x}_p$, $p \in \mathcal{P}$ are bounded, $\dot{\widetilde{x}}_{q^*} = \dot{x}_{q^*} - \dot{x}$ is bounded as well. We also know that $\widetilde{x}_{q^*} \in L^2$. Using Fact 3, we conclude that $\widetilde{x}_{q^*} \to 0$. Similarly, $\widetilde{y}_{q^*}$, $\widetilde{z}_{q^*} \to 0$. Therefore, we have $\mathbf{x}_{\mathrm{i}} \to 0$, and consequently $\mathbf{x}_{\mathrm{u}} \to 0$ as needed. $\square$

4 State-sharing

In the previous section we have relied on the fact that the set $\mathcal{P}$ was finite, so that the systems of estimator equations (7) and performance signal equations (12) were finite-dimensional. If the set $\mathcal{P}$ is infinite, a different approach is required. One alternative is to replace the individual estimator equations by a single system and use it to generate the estimation errors and performance signals, in other words, make the estimators "share" the same state. This idea in fact leads to a more efficient way of designing a supervisory control system even for the case when $\mathcal{P}$ has a finite but large number of elements. What makes such state sharing possible in our case is the fact that the control functions $f_p(\mathbf{x}_\mathrm{u}, \mathbf{w})$ and $g_p(\mathbf{x}_\mathrm{u}, \mathbf{w})$ are *separable*, i.e., of the form $a^T(p)b(\mathbf{x}_\mathrm{u}, \mathbf{w})$.

Consider the equations

$$\begin{aligned}
\dot{\widehat{x}}_1 &= -\widehat{x}_1 + x & \dot{\widehat{y}}_2 &= -\widehat{y}_2 + w_2 \\
\dot{\widehat{x}}_2 &= -\widehat{x}_2 + w_1 & \dot{\widehat{z}}_1 &= -\widehat{z}_1 + z \\
\dot{\widehat{x}}_3 &= -\widehat{x}_3 - w_2\frac{xy+z}{2} & \dot{\widehat{z}}_2 &= -\widehat{z}_2 - yw_1 \\
\dot{\widehat{y}}_1 &= -\widehat{y}_1 + y & \dot{\widehat{z}}_3 &= -\widehat{z}_3 + xw_2 + yw_2\frac{xy+z}{2}
\end{aligned} \tag{20}$$

with x, y and z given by (2). If we now define

$$\begin{aligned}
x_p &:= \widehat{x}_1 + p_1\widehat{x}_2 + p_2\widehat{x}_3 \\
y_p &:= \widehat{y}_1 + p_2\widehat{y}_2 \\
z_p &:= \widehat{z}_1 + p_1\widehat{z}_2 + p_2\widehat{z}_3
\end{aligned} \tag{21}$$

then it is easy to check that x_p, y_p and z_p satisfy the original estimator equations (7). We can thus define the p-th estimation error $\mathbf{e}_p = (\widetilde{x}_p, \widetilde{y}_p, \widetilde{z}_p)^T$ by

$$\begin{aligned}
\widetilde{x}_p &:= \widehat{x}_1 + p_1\widehat{x}_2 + p_2\widehat{x}_3 - x \\
\widetilde{y}_p &:= \widehat{y}_1 + p_2\widehat{y}_2 - y \\
\widetilde{z}_p &:= \widehat{z}_1 + p_1\widehat{z}_2 + p_2\widehat{z}_3 - z
\end{aligned}$$

and use (20)–(21) in place of (7).

The performance signals can be obtained similarly. As for the candidate control laws and the switching logic, they can be defined exactly as before. One can check that the minimization procedure used in the switching algorithm reduces to that of finding roots of polynomials in one variable of degree at most 5, so its computational tractability is not an issue.

The above discussion explains how to *implement* the supervisory control algorithm in the case when the parameter set $\mathcal{P}$ is infinite. Once this is done, the question that arises is how to *prove* that this algorithm indeed performs the required task. Unfortunately, the switching cannot be guaranteed to stop in this case, and the problem of proving the state convergence remains open. Perhaps even more importantly, the same issue also arises when disturbances, noise, and/or unmodeled dynamics are present in the system.

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