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Robust stability conditions for switched linear systems: commutator bounds and the Łojasiewicz inequality

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Abstract— **This paper discusses conditions for stability of switched linear systems under arbitrary switching, formulated in terms of smallness of appropriate commutators of the matrices generating the switched system. Such conditions provide robust variants of well-known stability conditions requiring these commutators to vanish and leading to the existence of a common quadratic Lyapunov function. The main contribution of the paper is to apply the Łojasiewicz inequality to characterize the persistence of a common quadratic Lyapunov function as the matrices are perturbed so that their commutators no longer vanish but instead are sufficiently small. It is shown how known constructions of common quadratic Lyapunov functions for commuting matrices and for matrices generating nilpotent or solvable Lie algebras can be used, in conjunction with the Łojasiewicz inequality, to estimate allowable deviations of the commutators from zero.**

I. Introduction

A *switched system* is defined by a family of individual systems (modes) and a piecewise constant switching signal that determines the active mode at each time instant [1]. The fact that commutation relations between the constituent modes play an important role in determining stability of the switched system under arbitrary switching, as well as the existence of a common Lyapunov function, is by now well known. These issues were investigated for linear switched systems in [2], [3], [4], [5], [6], [7] and for nonlinear switched systems in [8], [9], [10], [11], [12].

Conditions requiring certain commutators to vanish are not robust with respect to arbitrarily small perturbations to the system data, and as such are not very useful for practical purposes. Our recent work in [13] has focused on developing more robust stability criteria for switched linear systems, which require the generating matrices to be sufficiently close to commuting or to generating a solvable or "solvable plus compact" Lie algebra. For discrete-time switched linear systems, we were able to derive an upper bound on the norm of the commutators under which the system is still exponentially stable. For continuous-time switched linear systems, stability conditions were formulated as bounds on the size of suitable components in Levi or Cartan decompositions of the Lie algebra. Related work in [14], [15] examines the problem of choosing state feedback matrices that yield approximate simultaneous triangularizability in closed loop (i.e., give closed-loop matrices whose associated Lie algebra is approximately solvable).

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In this paper we investigate a more direct approach to robustifying the stability conditions based on commutators for continuous-time switched linear systems, by bringing in an idea not previously used in this context. Given a family of matrices whose commutators are small, we can ask the following question: is this family of matrices necessarily close to a family of commuting matrices? More generally, if the Lie algebra generated by given matrices is approximately nilpotent or solvable (in the sense that certain commutators almost vanish), is this family of matrices necessarily close to another family which generates a nilpotent or solvable Lie algebra? The answer to these questions is "yes" provided the matrices are drawn from a compact set (as we already noted in [13, Remark 1]). The main new idea proposed here is to gain a better understanding of this issue by using the *Łojasiewicz inequality* [16]. Given a function (usually a polynomial or a maximum of polynomials) which takes a small value at a given point, the Łojasiewicz inequality provides an upper bound on the distance from this point to the set of zeros of the function. What this result allows us to do in the present context is, given an upper bound on the size of commutators of given matrices, estimate the distance from this family of matrices to a family of commuting matrices. If this distance is small enough, then standard perturbation arguments can be used to show that a common quadratic Lyapunov function for the latter, commuting matrix family (which can be constructed by known tools) still serves as a common Lyapunov function for the original matrix family. We therefore obtain a robust stability criterion in terms of the size of the commutators. For matrices generating almost nilpotent or almost solvable Lie algebras, the reasoning is very similar but involves higher-order commutators.

The above program is carried out in the paper as follows. In Section II we set up the problem and discuss the Łojasiewicz inequality and its immediate consequences. In Section III we conduct a perturbation analysis and establish a sufficient condition for stability (Proposition 1) that relates the size of the commutators to the eigenvalues of the matrices provided by the common quadratic Lyapunov function and the constants appearing in the Łojasiewicz inequality. A more detailed analysis of these parameters follows in Sections IV and V, where specific known constructions of common quadratic Lyapunov functions for commuting matrices (Section IV-A) and matrices generating a solvable Lie algebra (Section IV-B) are utilized to obtain more explicit estimates. A summary and discussion of open questions in Section VI conclude the paper.

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II. PRELIMINARIES

Let $\{A_1, A_2, \ldots, A_N\} \subset \mathbb{R}^{n\times n}$ be a finite set of Hurwitz matrices generating the switched linear system

$$\dot{x} = A_\sigma x \tag{1}$$

where $\sigma : [0,\infty) \to \{1,\ldots,N\}$ is a piecewise constant switching signal.

Suppose that for all $i, j \in \{1,\ldots,N\}$ the corresponding commutator $[A_i, A_j]$ satisfies the bound

$$\|[A_i, A_j]\|_F \le \varepsilon \tag{2}$$

where ε is a small positive number and $\|\cdot\|_F$ denotes the Frobenius matrix norm (which is the Euclidean norm of the corresponding vector in $\mathbb{R}^{n^2}$, $\|A\|_F^2 := \sum_{k,l=1}^n A_{kl}^2$).

Viewed as a function on $\mathbb{R}^{2n^2}$, the square of the commutator norm $\|[\cdot,\cdot]\|_F^2$ is a polynomial of degree 4. We need to combine these squared norms for each pair (A_i, A_j) into a single function. We can do this by defining

$$f(A_1, A_2, \ldots, A_N) := \sum_{1\le i<j\le N} \|[A_i, A_j]\|_F^2$$

or

$$f(A_1, A_2, \ldots, A_N) := \max_{1\le i<j\le N} \|[A_i, A_j]\|_F^2$$

The first is itself a polynomial in the elements of all the matrices, the second is the maximum of a finite number of polynomials. In either case, the Łojasiewicz inequality applies to the function f (the original inequality applies to real analytic functions on compact sets, and more specific results have since been obtained for polynomials and for maxima of finitely many polynomials; see [16] and the references therein). The Łojasiewicz inequality stipulates the existence of two positive constants[1] C and α such that

$$Cf(A_1, A_2, \ldots, A_N) \ge \delta^\alpha \tag{3}$$

where δ is the distance from the N-tuple of matrices $(A_1, \ldots, A_N)$ to the set of N-tuples of commuting matrices (this distance being defined by stacking the matrices together, computing the difference element-wise, and taking the Frobenius norm). In view of (2), this implies that there exist matrices B_i, $i = 1,\ldots,N$ such that for all $i, j \in \{1,\ldots,N\}$ we have:

1) $[B_i, B_j] = 0$
2) $A_i = B_i + \Delta_i$ with $\|\Delta_i\|_F \le \delta := (C\ell\varepsilon^2)^{1/\alpha}$

where ℓ equals either $N(N-1)/2$ or 1 depending on whether f is defined as a sum or as a max. From now on we assume the latter and drop ℓ.

Instead of (2) we can place bounds on higher-order commutators, e.g.,

$$\|[A_i, [A_j, A_k]]\|_F \le \varepsilon$$

for all $i, j, k \in \{1,\ldots,N\}$. Then the above construction can be adapted in the obvious way and we have the existence of matrices B_i which are no longer commuting but generate a Lie algebra that is nilpotent of the corresponding order. More generally, a Lie algebra is solvable if appropriate (but not all) high-order commutators vanish, and so by placing bounds on these commutators of the form

$$\|[[A_i, A_j], ..., [A_k, A_\ell]]\|_F \le \varepsilon$$

for all $i, j, k, \ell \in \{1,\ldots,N\}$, we have the existence of matrices B_i which generate a solvable Lie algebra.

[1] See Section V for more information about these constants.

III. PERTURBATION ANALYSIS

If ε is small enough then δ defined in item 2 above is small enough so that the matrices B_i, $i = 1,\ldots,N$ are still Hurwitz. Since the matrices B_i commute (or, more generally, generate a nilpotent or solvable Lie algebra), we then know that they possess a common quadratic Lyapunov function and generate an exponentially stable switched linear system. The idea then is to do perturbation analysis, to show that the switched linear system generated by the original matrices A_i is stable as well.

Let $V(x) = x^T P x$ be the common Lyapunov function for the B_i's, so that

$$PB_i + B_i^T P = -Q_i < 0 \qquad \forall\, i = 1,\ldots,N \tag{4}$$

The derivative of the same Lyapunov function V along the flows given by the original matrices A_i is characterized by

$$\begin{aligned} PA_i + A_i^T P &= P(B_i + \Delta_i) + (B_i + \Delta_i)^T P \\ &= \underbrace{PB_i + B_i^T P}_{=-Q_i} + P\Delta_i + \Delta_i^T P \end{aligned}$$

and this is still negative definite if Δ_i is small enough in a suitable sense. The following sufficient condition for this is derived in [17, p. 342] and [1, p. 42]:

$$\|\Delta_i\|_2 < \frac{\lambda_{\min}(Q_i)}{2\lambda_{\max}(P)} \tag{5}$$

where $\|\cdot\|_2$ is the matrix norm induced by the Euclidean norm and $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the smallest and the largest eigenvalue of a symmetric matrix, respectively.

To express this in terms of the Frobenius norm that we were using earlier, we utilize the following easily proved fact [18, Lemma 4.9]:

$$\|\Delta_i\|_2 \le \|\Delta_i\|_F$$

Putting this together with the bound on $\|\Delta_i\|_F$ from the previous section, we obtain the bound

$$(C\varepsilon^2)^{1/\alpha} < \frac{\lambda_{\min}(Q_i)}{2\lambda_{\max}(P)}$$

and can state the following result.

Proposition 1 *The switched system* (1) *is exponentially stable, with common quadratic Lyapunov function $x^T P x$, if*

$$\varepsilon^2 < \frac{1}{C}\left(\frac{\lambda_{\min}(Q_i)}{2\lambda_{\max}(P)}\right)^\alpha \tag{6}$$

where ε comes from (2)*, C and α come from* (3)*, P and Q_i, $i = 1, \ldots, N$ come from* (4)*, and the matrices B_i, $i = 1, \ldots, N$ satisfy conditions 1 and 2 of the previous section.*

To have a constructive upper bound on ε, we need to get a handle on the eigenvalues of the matrices P and Q_i from the Lyapunov equations (4) as well as on the constants C and α from the Łojasiewicz inequality (3). These are discussed next.

IV. More on Lyapunov matrices

When Hurwitz matrices B_i, $i = 1, \ldots, N$ commute or generate a nilpotent or solvable Lie algebra, a common quadratic Lyapunov function for this family of matrices can be constructed explicitly by using the methods of [4] or [6], respectively. We start with the construction from [4] for the commuting case.

A. Commuting matrices

The construction from [4] applies when our commuting matrix family is finite, say $B_1, B_2, \ldots, B_N$, and it consists in solving the following sequence of Lyapunov equations:

$$\begin{aligned} P_1 B_1 + B_1^T P_1 &= -I \\ P_2 B_2 + B_2^T P_2 &= -P_1 \\ &\vdots \\ P_N B_N + B_N^T P_N &= -P_{N-1} \end{aligned} \tag{7}$$

Then $V(x) = x^T P_N x$ is the desired common quadratic Lyapunov function for all N linear systems. The matrices P_i are given explicitly by the integral expressions

$$P_1 = \int_0^\infty e^{B_1^T t} e^{B_1 t} dt, \;\; P_2 = \int_0^\infty e^{B_2^T t} P_1 e^{B_2 t} dt, \;\ldots$$

Let M be a common bound on the induced 2-norm of the B_i's:

$$\|B_i\|_2 \le M, \qquad i = 1, \ldots, N \tag{8}$$

An estimate on this number M can be easily obtained by slightly enlarging $\max_{1\le i\le N} \|A_i\|_2$. Since the B_i's are Hurwitz, there also exist positive constants c and λ such that

$$\|e^{B_i t}\|_2 \le c e^{-\lambda t} \qquad \forall\, t \ge 0, \; i = 1, \ldots, N \tag{9}$$

(taking c and λ to be the same for all i is always possible, but we could also work with different ones to achieve more precise results). This gives the lower and upper bounds

$$e^{-Mt}|x| \le |e^{B_i t} x| \le c e^{-\lambda t}|x| \qquad \forall\, x$$

(see [17, pp. 107 and 159]). Plugging these into the integral expression for P_1, we obtain the lower bound

$$\begin{aligned} x^T P_1 x &= \int_0^\infty |e^{B_1 t} x|^2 dt \\ &\ge \int_0^\infty e^{-2Mt} dt\, |x|^2 = \frac{1}{2M}|x|^2 \end{aligned}$$

and the upper bound

$$x^T P_1 x \le c^2 \int_0^\infty e^{-2\lambda t} dt\, |x|^2 = \frac{c^2}{2\lambda}|x|^2$$

valid for all x. At the next step, we can use these bounds to obtain bounds for P_2:

$$\begin{aligned} x^T P_2 x &\ge \frac{1}{2M} \int_0^\infty |e^{B_2 t} x|^2 dt \\ &\ge \frac{1}{2M} \int_0^\infty e^{-2Mt} dt\, |x|^2 = \frac{1}{4M^2}|x|^2 \end{aligned}$$

and, similarly,

$$x^T P_2 x \le \frac{c^4}{4\lambda^2}|x|^2$$

Proceeding in this way, we conclude that

$$\lambda_{\min}(P_i) \ge \frac{1}{(2M)^i}, \qquad \lambda_{\max}(P_i) \le \frac{c^{2i}}{(2\lambda)^i} \tag{10}$$

for $i = 1, \ldots, N$. Now we can use these bounds to derive a lower bound for the stability margin $\lambda_{\min}(Q_i)/(2\lambda_{\max}(P))$ appearing in (6). Since the order in which the equations (7) are solved does not matter, we can match the Lyapunov equation (4) for each i with the last equation in (7) which means setting $P := P_N$ and $Q_i := P_{N-1}$. Then (10) gives

$$\frac{\lambda_{\min}(Q_i)}{2\lambda_{\max}(P)} \ge \frac{\lambda^N}{M^{N-1}c^{2N}}$$

and a sufficient condition for (6) to hold is

$$\varepsilon^2 < \frac{1}{C}\left(\frac{\lambda^N}{M^{N-1}c^{2N}}\right)^\alpha \tag{11}$$

For this bound to be useful, we need to have some estimates of the overshoot constant c and the exponential decay rate λ appearing in (9). However, the matrices B_i are unknown, so we can realistically expect to only know the corresponding quantities for the given matrices A_i—let us call them $\bar{c}$ and $\bar{\lambda}$—and then need to estimate them for the B_i's. There are several ways to do this. One is to look at the characteristic polynomial of A_i, check by how much its value can change if the matrix elements are perturbed by δ, and use the Łojasiewicz inequality again to estimate the distance between the roots of the original and the perturbed polynomial; this allows us to estimate λ because λ corresponds to the least stable eigenvalue. Another way is to connect the behavior of solutions of the two linear systems $\dot{x} = A_i x$ and $\dot{x} = B_i x = (A_i - \Delta_i)x$ by using small-gain analysis and derive time-domain convergence estimates for the latter system from those of the former. A third way, and perhaps the cleanest, is to use Lyapunov analysis, essentially in the opposite way to how we proceeded from (4). Suppose that we know positive definite matrices $\bar{P}_i$ and $\bar{Q}_i$ which satisfy the Lyapunov equation

$$\bar{P}_i A_i + A_i^T \bar{P}_i = -\bar{Q}_i$$

($\bar{P}_i$ can be different for each i, so we are not talking about a common Lyapunov function here). From this, the overshoot $\bar{c}$ and decay rate $\bar{\lambda}$ for A_i are given by

$$\bar{c} = \frac{\lambda_{\max}(\bar{P}_i)}{\lambda_{\min}(\bar{P}_i)}, \qquad \bar{\lambda} = \frac{\lambda_{\min}(\bar{Q}_i)}{\lambda_{\max}(\bar{P}_i)}$$

Suppose that δ is small enough so that

$$2\delta\lambda_{\max}(\bar{P}_i) \leq \gamma\lambda_{\min}(\bar{Q}_i)$$

for some $\gamma \in (0,1)$. Then we have

$$\bar{P}_i B_i + B_i^T \bar{P}_i = \bar{P}_i(A_i - \Delta_i) + (A_i - \Delta_i)^T \bar{P}_i \leq (1-\gamma)\bar{Q}_i$$

and it follows that the overshoot c for B_i equals $\bar{c}$ and the decay rate λ for B_i equals $(1-\gamma)\bar{\lambda}$. This, by the way, automatically ensures that B_i is Hurwitz.

B. Nilpotent or solvable Lie algebra

For the case of higher-order nilpotency or solvability, the above construction of a common Lyapunov function does not apply and only the construction from [6] is available. This construction relies on Lie's theorem (see, e.g., [19, §9.2]) which says that if the matrices B_i, $i = 1,\dots,N$ generate a solvable Lie algebra, then they have a common invariant flag and, hence, there exists a unitary coordinate transformation matrix that simultaneously brings them to the upper triangular form.[2] Since this unitary transformation preserves the eigenvalues appearing in the formula (6), we can assume that the B_i's are already upper triangular. Note that the B_i's are in general complex-valued, and so will be the Lyapunov function in the original basis, but this has no consequence for our results. Suppose that we have, as in the previous subsection, positive numbers M and λ such that the bounds (8) and (9) hold, i.e., M and $-\lambda$ are upper bounds on the induced 2-norm of the B_i's (and hence on the absolute values of their individual elements) and on the real parts of their eigenvalues (diagonal elements), respectively. The result of [6] tells us that we can look for a real-valued matrix $P = P^T > 0$ solving $PB_i + B_i^* P = -Q_i < 0$ in the diagonal form $P = \mathrm{diag}\{p_1,\dots,p_n\}$. For concreteness, let us say that we want $Q_i \geq I$ for each i. This is guaranteed if the hermitian matrices $-PB_i - B_i^* P - I$, $i = 1,\dots,N$ are all positive definite, which is in turn ensured if their principal minors are all positive. We can satisfy this property by choosing the numbers p_i iteratively, following the procedure given in [6] (with very minor modifications). Namely, fix a small positive number $\rho > 0$, choose $p_1 := (1+\rho)/(2\lambda)$, and then for $k = 1,\dots,n-1$ choose

$$p_{k+1} := \frac{k!k2^{k-1}M^{k+1}\max_{1\leq i\leq k} p_i^{k+1} + 2\rho}{2\lambda\rho}$$

Using the same counting argument as in [6], it is not hard to check that all principal minors of $-PB_i - B_i^* P - I$ will then be no smaller than ρ. By construction, we have $\lambda_{\min}(Q_i) \geq 1$ and $\lambda_{\max}(P) = \max_{1\leq i\leq n} p_i$. Therefore, the bound (6) is satisfied if

$$\varepsilon^2 < \frac{1}{C(2\max_{1\leq i\leq n} p_i)^{\alpha}} \tag{12}$$

[2] The coordinate transformation is in general complex. The orthonormality of the basis in which the matrices become triangular is not part of the usual statement of Lie's theorem, but it can always be achieved by the Gramm-Schmidt process.

V. More on Łojasiewicz constants

The constants C and α depend on the compact set from which the matrices A_i are drawn, as well as on the number N of these matrices; once the matrices are fixed (and, in the nilpotent or solvable case, the desired commutators are fixed), these constants are also fixed.

As discussed in [16], for maxima of polynomial functions of given degree some explicit bounds on the exponent α in (3) are available. Our f is a polynomial of degree 4, or a maximum of finitely many polynomials of degree 4, in the elements of the matrices A_i. So, in view of the explicit bounds given in [16] we can view the exponent α as something that we can easily estimate in practice.

Explicit bounds on the constant C, on the other hand, are harder to come by in the literature as one tends to be interested in asymptotics only. Some information about C can be obtained by following the proofs given in [16]. Basically, C is given by the product of two terms. The first term is computed from the coefficients of the polynomials (expressed in a suitable basis). The second term depends on the size of the compact set to which the elements of the matrices A_i belong, as well as on α.

VI. Summary and discussion

We derived upper bounds on commutators which guarantee that the switched linear system (1) is exponentially stable and possesses a common quadratic Lyapunov function. We first established the basic bound given by the inequality (6), and then we used particular known constructions of a common Lyapunov function for commuting matrices and for matrices generating a solvable Lie algebra to arrive at the more specific bounds (11) and (12). There are three different ways to think about these results:

Qualitative These results confirm that the switched system is stable if the matrices "nearly" commute, or if they generate a "nearly" nilpotent or solvable Lie algebra. Even as a qualitative result this is of interest. The only such explicit result in the literature that we know is the one in [13, Section 2] given for discrete time and derived using direct calculations. (Of course a discrete-time version of the above analysis is also possible, based on replacing (5) by a suitable discrete-time version.) The continuous-time results in [13, Section 3] are different in nature and rely on the structure of the Lie algebra.

Asymptotic The results tell us that asymptotically as the stability margin $\lambda_{\min}(Q_i)/(2\lambda_{\max}(P))$ shrinks to 0, the admissible size of the commutator shrinks as this stability margin raised to the power of $\alpha/2$. This quantitative information is obtained from the knowledge of α only and does not depend on C.

Quantitative As we said in Section V, specific expressions for α in terms of the degree of the polynomials (which in the case of first-order commutators equals 4) are available in the literature. Getting a handle on C is a bit more problematic but can also be done, as mentioned in Section V. As for the matrices P and Q_i, we gave some explicit estimates for them in Section IV. So, in principle, for a given collection of matrices A_i we can calculate whether their commutators are small enough for our results to guarantee stability. On the other hand, for numerical purposes one can simply check feasibility of the LMIs

$$PA_i + A_i^T P < 0, \qquad i = 1, \dots, N$$

and this can almost certainly be done more efficiently than using the estimates we discussed. So, the primary interest of our results probably lies in the qualitative insight that they provide rather than in their potential to be used for calculations.

Some open avenues for future research are: comparing the above perturbation approach with the direct approach from [13, Section 2]; handling compact but not finite sets of matrices; and seeing if the Łojasiewicz inequality can yield any result for nonlinear (e.g., polynomial) switched systems.

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