Passification-based adaptive control with quantized measurements ⋆

Anton Selivanov * **Alexander Fradkov** *,** **Daniel Liberzon** ***

* *Saint Petersburg State University, St. Petersburg, Russia (e-mail: antonselivanov@gmail.com).*
** *Institute for Problems of Mechanical Engineering, Russian Academy of Sciences, St. Petersburg, Russia (e-mail: fradkov@mail.ru).*
*** *University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: liberzon@uiuc.edu).*

Abstract: We propose and analyze passification-based adaptive controller for linear uncertain systems with quantized measurements. Since the effect of the quantization error is similar to the effect of a disturbance, the adaptation law with σ-modification is used. To ensure convergence to a smaller set, the parameters of the adaptation law are being switched during the evolution of the system and a dynamic quantizer is used. It is proved that if the quantization error is small enough then the proposed controller ensures convergence of the state of a hyper-minimum-phase system to an arbitrarily small vicinity of the origin. Applicability of the proposed controller to polytopic-type uncertain systems and its efficiency is demonstrated by the example of yaw angle control of a flying vehicle.

1. INTRODUCTION

Adaptive control plays an important role in the real world problems, where exact system parameters are often unknown. One of the possible methods for adaptive control synthesis is *passification method* (Andrievskii and Fradkov (2006)). Starting from the works Fradkov (1974, 1976) this method proved to be very efficient and useful. Nevertheless, while implementing passification-based adaptive control, several issues may arise. First of all, disturbances inherent in most systems can cause infinite growth of the control gain. This issue may be overcome by introducing the so-called "σ-modification" (Lindorff and Carroll (1973); Ioannou and Kokotovic (1984)). Secondly, the measurements can experience time-varying unknown delay. This problem has been recently studied in Selivanov et al. (2013). The objective of this paper is to design and investigate passification-based adaptive control with quantized measurements.

Control with limited information has attracted growing interest in the control research community lately, largely motivated by the control over networks paradigm (Wong and Brockett (1997, 1999); Brockett and Liberzon (2000); Matveev and Savkin (2004)). Since the capacity of a communication channel is limited, sensor signals are digitized before being sent. Additional constraints can be imposed by defects of sensors. Both communication constraints and limited sensing capabilities can be modeled by quantization (Liberzon (2009)).

Although adaptive control of uncertain systems received considerable interest and has been widely investigated, there are few works devoted to adaptive control with quantized measurements. In Fradkov and Andrievsky (2008) the performance of an adaptive observer-based chaotic synchronization system under information constrains has been analyzed. A binary coder-decoder scheme has been proposed and studied in Fradkov et al. (2009) for synchronization of passifiable Lurie systems via limited-capacity communication channel. In Hayakawa et al. (2009) a direct adaptive control framework for systems with *input* quantizers has been developed. In Vu and Liberzon (2012) supervisory control scheme for uncertain systems with quantized measurements has been proposed. In supervisory control schemes only a finite family of candidate controllers is employed together with an estimator-based switching logic to select the active controller at every time.

Differently from these works, the control scheme proposed here does not require any observer and the quantizer has the general form. Unlike Vu and Liberzon (2012) we consider adaptive tuning of the controller gain, rather then switching between several known controllers. At the same time, to ensure convergence to a smaller set, our controller switches parameters of the adaptation law (for details see (5), (11)-(13)).

2. SYSTEM DESCRIPTION

Consider a linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}^l$ is the output, unknown matrices A, B, and C have appropriate dimensions. Following Andrievskii and Fradkov (2006) we introduce the notion of *hyper-minimum-phase* (HMP) systems.

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Definition 1. For a given $g \in \mathbb{R}^l$ the transfer function $g^T W(s) = g^T C(sI - A)^{-1} B$ is called *hyper-minimum-phase* (HMP) if the polynomial $g^T W(s) \det(sI - A)$ is Hurwitz and $g^T CB > 0$.

Further we will assume that unknown matrices A, B, and C belong to some known compact set of uncertainties Ξ and we know some $g \in \mathbb{R}^l$ such that for all $(A, B, C) \in \Xi$ the function $g^T W(s) = g^T C(sI - A)^{-1} B$ is HMP.

Note that if there exists g such that $g^T W(s)$ is HMP then $\|g\|^{-1} g^T W(s)$ is also HMP. Therefore, without loss of generality, we assume that $\|g\| = 1$.

2.1 Passification lemma

Lemma 1. (Passification lemma). For existence of a matrix $P > 0$ and a number $\varkappa > 0$ such that

$$PA_\varkappa + A_\varkappa^T P < 0, \quad PB = C^T g,$$

where $A_\varkappa = A - B\varkappa g^T C$, it is necessary and sufficient that $g^T W(s) = g^T C(sI - A)^{-1} B$ is HMP.

Proof. See Fradkov (1974, 2003).

Remark 2. If the transfer function $g^T W(s) = g^T C(sI - A)^{-1} B$ is HMP then there exists $\varkappa > 0$ such that the control law $u = -\varkappa \tilde{y} + v$, where v is a new control signal, makes the system $\dot{x}(t) = Ax(t) + Bu(t)$, $\tilde{y}(t) = g^T Cx(t)$ *strictly passive* with respect to a new input v, i. e. there exists a nonnegative scalar function $V(x)$ and a scalar function $\varphi(x)$, where $\varphi(x) > 0$ for $x \neq 0$, such that

$$V(x) \leq V(x_0) + \int_0^t \left[\tilde{y}^T(t) v(t) - \varphi(x(t))\right] dt$$

for any solution satisfying $x(0) = x_0$. Appropriate value for $\varkappa$ is any positive number such that

$$\varkappa > \varkappa_0 = -\inf_{\omega \in \mathbb{R}} \operatorname{Re}\left\{g^T W(i\omega)^{-1}\right\}. \tag{2}$$

Note that if $g^T W(s)$ is HMP and $\lambda < 0$ is the maximum real part of its zeros then for any $\alpha \in (0, -2\lambda)$ the function $g^T W_\alpha(s) = g^T W(s - \frac{\alpha}{2}) = g^T C(sI - A_\alpha)^{-1} B$, where $A_\alpha = A + \frac{\alpha}{2} I$, is also HMP. Therefore there exists $P > 0$ and $\varkappa > 0$ such that $PA_0 + A_0^T P < 0$ and $PB = C^T g$ with $A_0 = A + \frac{\alpha}{2} I - B\varkappa g^T C$. Thus, the following corollary is true.

Corollary 3. If $g^T C(sI - A)^{-1} B$ is HMP, $\lambda < 0$ is the maximum real part of its zeros, and $\alpha \in (0, -2\lambda)$, then there exists a matrix $P > 0$ and a number $\varkappa > 0$ such that

$$PA_\varkappa + A_\varkappa^T P + \alpha P < 0, \quad PB = C^T g, \tag{3}$$

where $A_\varkappa = A - B\varkappa g^T C$.

2.2 Dynamic quantization

In the remaining part of the paper we assume that the controller receives quantized measurements. Suppose that $\|y(0)\| \leq M$, where M is a known positive number. Following Liberzon (2009) we introduce a *quantizer with the quantization range M and the quantization error bound* Δ as a mapping $q \colon y \mapsto q(y)$ from $\mathbb{R}^l$ to a finite subset of $\mathbb{R}^l$ such that

$$\|y\| \leq M \Rightarrow \|q(y) - y\| \leq \Delta.$$

We will refer to the quantity $e = q(y) - y$ as the *quantization error*. By *dynamic quantizer* we will mean the mapping

$$q_\mu(y) = \mu q\left(\frac{y}{\mu}\right), \tag{4}$$

where $\mu > 0$. The quantization range for this quantizer is μM and the quantization error bound is $\Delta_\mu = \mu\Delta$. We can think of μ as the "zoom" variable: increasing μ corresponds to zooming out and essentially obtaining a new quantizer with larger quantization range and quantization error bound, whereas decreasing μ corresponds to zooming in and obtaining a quantizer with a smaller quantization range but also a smaller quantization error bound.

2.3 Adaptive algorithm structure

Suppose we know some $g \in \mathbb{R}^l$, $\|g\| = 1$ for which $g^T W(s) = g^T C(sI - A)^{-1} B$ is HMP for all $(A, B, C) \in \Xi$. We will consider the adaptive algorithm

$$\begin{aligned} u(t) &= -k(t) g^T q_{\mu(t)}(y(t)), \\ \dot{k}(t) &= \gamma [g^T q_{\mu(t)}(y(t))]^2 - a(t) k(t), \end{aligned} \tag{5}$$

where $\gamma > 0$, $a(t)$ is the switched piecewise constant regularization parameter, $q_{\mu(t)}$ is the switched (dynamical) quantizer with piecewise constant "zooming" parameter $\mu(t)$. Initial value of $k(t)$ is an arbitrary chosen number (usually $k(0) = 0$). The motivation for the use of such a controller is that the effect of the quantization error is similar to the effect of a disturbance. For disturbed systems a similar controller ensures ultimate boundedness of the trajectories (see Andrievskii and Fradkov (2006)), that is, all trajectories enter some compact set Ω in finite time. The size of Ω depends on the magnitude of disturbance ($\Delta_{\mu(t)}$ in our case) and the value of $a(t)$. The idea of the controller that is proposed here is as follows. We define a sequence of switching instants $t_0 < t_1 < t_2 < \ldots$ and choose such μ_0, a_0 that if $\mu(t) = \mu_0$, $a(t) = a_0$ for $t \in [t_0, t_1)$ then the system output $y(t)$ enters some compact set $\Omega_1 \subsetneq \{y \in \mathbb{R}^l | \, \|y\| \leq M\}$ on $[t_0, t_1)$. Then it becomes possible to choose $\mu(t) = \mu_1$ and $a(t) = a_1$ for $t \in [t_1, t_2)$ so that the output enters a smaller set $\Omega_2 \subsetneq \Omega_1$ on $[t_1, t_2)$. By repeating this tuning procedure we ensure convergence of the trajectory to some $\Omega_\infty = \bigcap_{i=1}^\infty \Omega_i$. Below the tuning procedure for $\mu(t)$ and $a(t)$ is described.

We recall that $g \in \mathbb{R}^l$ is such that for all $(A, B, C) \in \Xi$ the function $g^T W(s)$ is HMP. Let $\lambda < 0$ be the maximum real part of zeros of $g^T W(s)$ over all $(A, B, C) \in \Xi$, i. e.

$$\lambda = \max\{\operatorname{Re}(s) | g^T W(s) = 0, (A, B, C) \in \Xi\}. \tag{6}$$

Let us fix some $\alpha \in (0, -2\lambda)$. According to Corollary 3 and formula (2) for any $(A, B, C) \in \Xi$ and $\varkappa$ such that

$$\varkappa > -\inf_{\omega \in \mathbb{R}} \operatorname{Re}\left\{g^T W_\alpha(i\omega)^{-1}\right\}, \tag{7}$$

where $g^T W_\alpha(s) = g^T W(s - \frac{\alpha}{2})$, there exists $P > 0$ such that (3) are true. We choose $\varkappa_*$ such that

$$\varkappa_* > \sup_{(A,B,C) \in \Xi} \left(-\inf_{\omega \in \mathbb{R}} \operatorname{Re}\left\{g^T W_\alpha(i\omega)^{-1}\right\}\right), \tag{8}$$

which exists since Ξ is compact. Let us fix some $\gamma > 0$ and suppose there is a known estimate $V_0 > 0$ such that

$$x^T(0) P x(0) + \gamma^{-1}(k(0) - k_*)^2 \leq V_0,$$

where

$$k_* = \varkappa_* + \frac{\varkappa_*}{\sqrt{2}}. \quad (9)$$

Without loss of generality we assume that $x^T(0)Px(0) \le V_0 \Rightarrow \|y(0)\| \le M$, where M is a known quantization range. Otherwise we can take such μ_0 that $x^T(0)Px(0) \le V_0 \Rightarrow \|y(0)\| \le \mu_0 M$ and then redefine the quantizer $\tilde{q}(y) = \mu_0 q(\frac{y}{\mu_0})$. Denote

$$\rho_1 = \left(1 + \frac{1}{\sqrt{2}}\right)^2 \frac{\varkappa_*^2}{\gamma}, \quad \rho_2 = \left(3 + 2\sqrt{2}\right) \frac{2\varkappa_*}{\alpha}. \quad (10)$$

Let us fix some $\nu > 0$. This parameter will determine the frequency of switching. We choose instants for switching as

$$t_{i+1} = t_i + \frac{1}{\alpha} \ln \frac{V_i - \rho_1 - \rho_2 \frac{V_i}{V_0}\Delta^2}{\nu}, \quad t_0 = 0, \quad (11)$$

where V_i are calculated recursively by

$$V_{i+1} = \rho_1 + \rho_2 \frac{V_i}{V_0}\Delta^2 + \nu. \quad (12)$$

Further we will show that the logarithm in (11) is well defined and $t_{i+1} > t_i$. Meanwhile, it may happen so that $t_i \xrightarrow[i\to\infty]{} t_\infty < \infty$, i. e. when $i \to \infty$ the controller (5) has to switch infinitely often. To prevent this we assume that the switching stops when the value of V_i is small enough (see Remark 6).

In the next section we analyze adaptive controller (5) where the quantizer is defined by (4) and for $t \in [t_i, t_{i+1})$

$$\mu(t) = \mu_i = \sqrt{\frac{V_i}{V_0}},$$
$$a(t) = a_i = \alpha + \gamma\mu_i^2\Delta^2 \left(\frac{1}{\sqrt{\gamma\rho_1}} + \frac{\sqrt{2}}{\varkappa_*}\right). \quad (13)$$

3. MAIN RESULT

To prove the main result we will use the following Lyapunov-like function:

$$V(x,k) = x^T P x + \gamma^{-1}(k - k_*)^2. \quad (14)$$

Here k_* is determined by (9) with $\varkappa_*$ given by (8) and $P = P_{(A,B,C)}$ is such that (3) are valid with $\varkappa = \varkappa_*$. In what follows we assume that $x(0)$ belongs to some known compact set X. Though the values of $(A,B,C) \in \Xi$ are not given and, therefore, the value of $P_{(A,B,C)}$ is unknown, it is always possible to find an upper bound V_0 for $V(x(0),k(0))$ satisfying

$$x^T(0)P_{(A,B,C)}x(0) + \gamma^{-1}(k(0) - k_*)^2 \le V_0 \ \ \forall (A,B,C) \in \Xi, \quad (15)$$

since X and Ξ are known compact sets.

Theorem 4. Let there exist $g \in \mathbb{R}^l$, $\|g\| = 1$, such that $g^T W(s) = g^T C(sI - A)^{-1}B$ is HMP for all $(A,B,C) \in \Xi$ and $\lambda < 0$ is given by (6). For a fixed $\alpha \in (0, -2\lambda)$ let a number $\varkappa_* > 0$ satisfy (8). Suppose that a number V_0 is such that (15) is true and the quantization error bound Δ is such that $\Delta^2 < V_0\rho_2^{-1}$, where ρ_2 is defined in (10). Then for any $\varepsilon > 0$ there exists $\gamma > 0$ and $\nu > 0$ such that the controller (5) with the switching algorithm (11)-(13) ensures existence of such i that

$$\forall t \ge t_i \quad \|x(t)\| < \varepsilon.$$

Moreover, the tuning coefficient $k(t)$ is bounded for $t \ge 0$.

Lemma 5. For any positive scalars ρ_1, ρ_2, Δ, ν if V_0 is such that

$$\rho_1 + \rho_2\Delta^2 + \nu < V_0$$

then the sequence

$$V_{i+1} = \rho_1 + \rho_2\frac{V_i}{V_0}\Delta^2 + \nu$$

monotonically decreases to the value

$$V_\infty = \frac{\rho_1 + \nu}{1 - \rho_2\Delta^2 V_0^{-1}}.$$

Proof of Lemma 5 is induction on i. For $i = 0$ we have

$$V_1 = \rho_1 + \rho_2\Delta^2 + \nu < V_0.$$

Suppose that $i > 0$ and for $j < i$ it is proved that $V_j < V_{j-1}$. Then

$$V_i = \rho_1 + \rho_2\frac{V_{i-1}}{V_{i-2}}\frac{V_{i-2}}{V_0}\Delta^2 + \nu < \rho_1 + \rho_2\frac{V_{i-2}}{V_0}\Delta^2 + \nu = V_{i-1}.$$

Therefore V_i is a monotonically decreasing sequence of positive numbers, and so it has a limit value, which is a solution of the equation

$$V = \rho_1 + \rho_2\frac{V}{V_0}\Delta^2 + \nu,$$

i. e. $V = V_\infty$. ■

Proof of Theorem 4. Since $\Delta^2 < V_0\rho_2^{-1}$ and $\varkappa_*$, α, V_0 are fixed it is possible to find such $\gamma > 0$ and $\nu > 0$ that

$$\rho_1 + \rho_2\Delta^2 + \nu < V_0. \quad (16)$$

Then from Lemma 5 we have $V_i < V_{i-1}$, therefore $\mu_i < \mu_{i-1}$. Then

$$(V_i - \rho_1 - \rho_2\mu_i^2\Delta^2)\nu^{-1} = (\rho_2\Delta^2(\mu_{i-1}^2 - \mu_i^2) + \nu)\nu^{-1} > 1.$$

Consequently, $t_{i+1} > t_i$.

Denote $q_i(y) = \mu_i q(\frac{y}{\mu_i})$, $\Delta_i = \mu_i\Delta$ and calculate derivative of (14) along the trajectories of the system (1), (5) for $t \in [t_i, t_{i+1})$:

$$\begin{aligned}\dot{V} &= x^T[PA + A^TP]x - 2x^TPBkg^Tq_i(y) + \\ &2(k - k_*)(g^Tq_i(y))^2 - 2a_i\gamma^{-1}(k - k_*)k = \\ &x^T[PA + A^TP]x - 2x^TPBk_*g^Tq_i(y) - \\ &2x^TPB(k - k_*)g^Tq_i(y) + 2(k - k_*)(g^Tq_i(y))^2 - \\ &2a_i\gamma^{-1}(k - k_*)k.\end{aligned}$$

Denote $A_* = A - Bk_*g^TC$. Using the equality $PB = C^Tg$ from (3) and relation $e_i(t) = q_i(y(t)) - y(t)$ we obtain:

$$\begin{aligned}\dot{V} &= x^T[PA_* + A_*^TP]x - 2x^TPBk_*g^Te_i - \\ &2y^Tg(k - k_*)g^Tq_i(y) + 2(k - k_*)(g^Tq_i(y))^2 - \\ &2a_i\gamma^{-1}(k - k_*)k = x^T[PA_* + A_*^TP]x - 2k_*x^TC^Tgg^Te_i + \\ &2(k - k_*)e_i^Tgg^Tq_i(y) - 2a_i\gamma^{-1}(k - k_*)^2 - \\ &2a_i\gamma^{-1}(k - k_*)k_*.\end{aligned}$$

Suppose that

$$\forall t \in [t_i, t_{i+1})\colon x^T(t)Px(t) \le V_i. \quad (17)$$

Then $\|y(t)\| \le \mu_i M$ and therefore $\|e_i(t)\| \le \Delta_i$. In this case using easily verifiable inequality $2y^Tz \le y^TQy + z^TQ^{-1}z$ (for $Q > 0$) we estimate mixed products as follows (recall that $\|g\| = 1$)

$- 2k_*x^TC^Tgg^Te_i \le \sigma(g^TCx)^2 + \sigma^{-1}k_*^2\Delta_i^2,$
$2(k-k_*)e_i^Tgg^Tq_i(y) \le 2\Delta_i^2(k-k_*) + \sigma^{-1}\Delta_i^2(k-k_*)^2 +$
$\sigma(g^TCx)^2 \le k_*\Delta_i^2 + k_*^{-1}(k-k_*)^2\Delta_i^2 + \sigma^{-1}\Delta_i^2(k-k_*)^2 +$
$\sigma(g^TCx)^2,$
$- 2a_i\gamma^{-1}(k-k_*)k_* \le a_i\gamma^{-1}(k-k_*)^2 + a_i\gamma^{-1}k_*^2.$

Thus,

$$\dot V + a_iV - \beta_i \le x^T[PA_* + A_*^TP + 2\sigma C^Tgg^TC + a_iP]x + \sigma^{-1}k_*^2\Delta_i^2 + k_*\Delta_i^2 + \left(k_*^{-1} + \sigma^{-1}\right)(k-k_*)^2\Delta_i^2 + a_i\gamma^{-1}k_*^2 - \beta_i.$$

Define

$$\beta_i = \sigma^{-1}k_*^2\Delta_i^2 + k_*\Delta_i^2 + a_i\gamma^{-1}k_*^2.$$

Now we select $\sigma = \frac{\varkappa_*}{\sqrt{2}}$ and using representation $(k-k_*)^2 = \gamma V - \gamma x^TPx$ we obtain:

$$\dot V + \alpha V - \beta_i \le x^T[PA_* + A_*^TP + 2\sigma C^Tgg^TC + \alpha P]x = x^T[PA_{\varkappa_*} + A_{\varkappa_*}^TP + \alpha P]x \le 0.$$

By comparison principle (see Khalil (2002)) for $t \in [t_i, t_{i+1}]$:

$$V(x(t),k(t)) \le \left(V(x(t_i),k(t_i)) - \frac{\beta_i}{\alpha}\right)e^{-\alpha(t-t_i)} + \frac{\beta_i}{\alpha}.$$

Using formulas (9) and (13) for k_* and a_i one may check that

$$\frac{\beta_i}{\alpha} = \rho_1 + \rho_2\mu_i^2\Delta^2. \tag{18}$$

Further, by induction on i, we show that

$$V(x(t_i),k(t_i)) \le V_i. \tag{19}$$

For $t_0 = 0$ the inequality $V(x(0),k(0)) \le V_0$ is fulfilled by construction. Suppose $i \ge 1$ and for $j < i$ (19) is true. Then

$$V(x(t_i),k(t_i)) \le \left(V_{i-1} - \frac{\beta_{i-1}}{\alpha}\right)e^{-\alpha(t_i-t_{i-1})} + \frac{\beta_{i-1}}{\alpha}.$$

Using (11) and (18) we obtain the desired inequality.

Since for $t \in [t_i, t_{i+1}]$: $V(x(t),k(t)) \le V(x(t_i),k(t_i))$ and $V_{i+1} < V_i$ we finally have:

$$\forall t \ge t_i \quad V(x(t),k(t)) \le V_i.$$

Now we prove that (17) is always true. Suppose that t_* is the smallest nonnegative instant such that

$$\begin{aligned} &t_* \in [t_i, t_{i+1}), \quad V(x(t_*),k(t_*)) = V_i, \\ &\exists\, d > 0\colon \forall t \in (t_*, t_*+d) \quad V(x(t),k(t)) > V_i. \end{aligned} \tag{20}$$

Then for $t \in [0, t_*]$ all above estimates are true. In particular $\dot V(x(t_*),k(t_*)) \le -\alpha V(x(t_*),k(t_*)) + \beta_i = -\alpha V_i + \beta_i$. In case $t_* = 0$, $\dot V$ stands for the right derivative. Since $V_i = \rho_1 + \rho_2\mu_i^2\Delta^2 + \nu > \frac{\beta_i}{\alpha}$, we find that $\dot V(x(t_*),k(t_*)) \le 0$. This contradicts with the second part of (20). Since $x^T(t_*)Px(t_*) \le V(x(t_*),k(t_*))$, (17) and, therefore, all previous estimates are valid for $t \ge 0$.

Now note that, since $\varkappa_*$, α, and V_0 are fixed, it is possible to choose such $\gamma > 0$ and $\nu > 0$ that (16) is fulfilled and

$$\frac{\rho_1 + \nu}{1 - \rho_2\Delta^2V_0^{-1}} < \varepsilon^2\lambda_{\min}(P).$$

Then, according to Lemma 5, there exists such i that $V_i < \varepsilon^2\lambda_{\min}(P)$. Thus, for $t \ge t_i$

$$\lambda_{\min}(P)\|x(t)\|^2 \le V(x(t),k(t)) \le V_i < \varepsilon^2\lambda_{\min}(P).$$

Boundedness of $k(t)$ follows from the boundedness of $V(t)$. ■

Remark 6. In practice one should choose such γ and ν that

$$\frac{\rho_1 + \nu}{1 - \rho_2\Delta^2V_0^{-1}} < \varepsilon^2\Lambda,$$

where

$$\Lambda = \min_{(A,B,C)\in\Xi} \lambda_{\min}(P_{(A,B,C)}),$$

and the switching should be stopped when $V_i < \varepsilon^2\Lambda$. Then it follows from the proof of Theorem 4 that $V(x(t),k(t)) \le V_i$ for $t \ge t_i$ and, therefore, $\|x(t)\| < \varepsilon$ for $t \ge t_i$.

Remark 7. Results of Theorem 4 are applicable to polytopic-type uncertainties. If

$$A = \sum_{j=1}^M \mu_j(t)A^{(j)}, \quad 0 \le \mu_j(t) \le 1, \quad \sum_{j=1}^M \mu_j(t) = 1,$$

and for any A of this form the function $g^TW(s) = g^TC(sI-A)^{-1}B$ is HMP, then one can solve (3) simultaneously for all the M vertices $A^{(j)}$, applying the same decision variables P and $\varkappa$. This technique is demonstrated by an example in the next section.

4. EXAMPLE: YAW ANGLE CONTROL

In this section we demonstrate the applicability and efficiency of the proposed control algorithm by the example of yaw angle control. Under some simplifying assumptions, dynamics of the lateral motion of the aircraft can be described by the equations (Fradkov and Andrievsky (2011))

$$\begin{cases} \dot\beta(t) = r(t) + c_1\beta(t) + b_1u(t), \\ \dot r(t) = c_2\beta(t) + c_3r(t) + b_2u(t), \quad y(t) = \begin{pmatrix} r(t) \\ \psi(t) \end{pmatrix}, \\ \dot\psi(t) = r(t), \end{cases} \tag{21}$$

where $\psi(t)$, $r(t)$ are the yaw angle and the yaw rate, respectively, and $\beta(t)$ denotes the sideslip angle; $u(t)$ is the rudder angle; c_i, b_i denote the aircraft model parameters. Following Fradkov and Andrievsky (2011) we take $c_1 = 0.75$, $c_2 = 33$, $c_3 = 1.3$, $b_1 = 19/15$, $b_2 = 19$ and suppose that for some system parameters only upper and lower bounds are known, namely,

$$c_1 \in [0.1, 1.5], \quad c_2 \in [25, 40]. \tag{22}$$

For $g = \frac{1}{\sqrt{2}}(1,1)^T$ the transfer function

$$g^TW(s) = \frac{b_2s^2 + (b_1c_2 - b_2c_1 + b_2)s + b_1c_2 - b_2c_1}{s\sqrt{2}(s^2 - (c_1+c_3)s + c_1c_3 - c_2)}$$

is HMP for all c_1, c_2 from (22). Moreover, for $\alpha = 0.08$ the function $g^TW(s - \frac{\alpha}{2})$ is HMP on the set (22). Therefore (3) are feasible. Possible values of decision variables (rounded to the second decimal digit) are

$$P \approx \begin{pmatrix} 2.52 & -0.17 & -1.89 \\ -0.17 & 0.05 & 0.16 \\ -1.89 & 0.16 & 3.36 \end{pmatrix}, \quad \varkappa_* = 3.4.$$

These values of P and $\varkappa_*$ satisfy (3) (where $\varkappa = \varkappa_*$) for all $(A,B,C) \in \Xi$, where Ξ is determined by (22). We choose $\gamma = 50$ and using (9) calculate $k_* \approx 5.8$. Suppose that $\Delta = 0.1$, $k(0) = 0$, and $x(0) = (\beta(0), r(0), \psi(0))^T$ is such that $V(x(0),k(0)) \le V_0 = 30$. Then we have $\Delta^2 < V_0\rho_2^{-1}$. Choosing $\nu = 0.1$ we find that $V_\infty \approx 0.9268$, thus for $\epsilon = 0.2$ there should exist such $t_i \ge 0$ that $V(x(t),k(t)) \le V_\infty + \epsilon \approx 1.1268$ for $t \ge t_i$.

i	t_i	V_i	μ_i	a_i
0	0	30	1	0.374
1	68.7	5.73	0.437	0.136
2	115.15	1.72	0.239	0.097
3	140.53	1.06	0.188	0.090
4	149.76	0.95	0.178	0.089

Table 1. Parameters of switching: t_i — instants of switching, V_i — upper bound for $V(x(t), k(t))$ on $[t_i, t_{i+1})$, μ_i — zooming parameter, a_i — regularizing parameter.

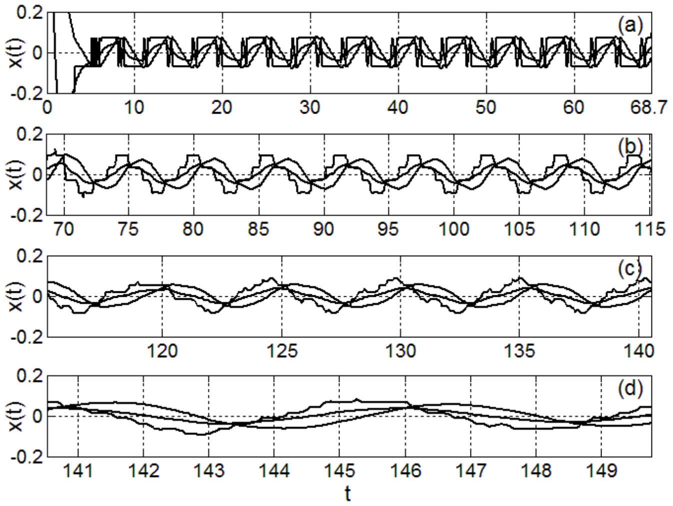

Fig. 1. Evolution of state: (a) – $t \in [t_0, t_1]$, (b) – $t \in [t_1, t_2]$, (c) – $t \in [t_2, t_3]$, (d) – $t \in [t_3, t_4]$.

Results of numerical simulations are presented in Figs. 1-3. The values of switched parameters are given in Table 1. Note that the intervals between consecutive switchings are decreasing, i. e. $t_{i+1} - t_i < t_i - t_{i-1}$. Therefore, to ensure convergence of V to a small vicinity of V_∞ the system should be able to switch fast enough. In our case $V_i < V_\infty + \epsilon$ after 3 switchings, that is for $t \geq t_3$, $V(x(t), k(t)) \leq V_\infty + \epsilon$. In Fig. 1 plots of $x(t)$ on each interval $[t_i, t_{i+1}]$ are depicted. One can notice that the trajectories are getting smoother. This happens due to the fact that the right side of (1), (5) gets "less discontinuous" when $\mu(t)$ decreases.

One may wonder why not to use $k(t) \equiv k_*$? In fact this is possible. The advantage of the adaptive control over the static one is that, while insuring the ultimate boundedness for the particular system of interest, adaptive controller results in a smaller control gain. In our case $\overline{\lim}_{t\to\infty} k(t) < 1.2$, while $k_* \approx 5.8$.

5. CONCLUSION

For hyper-minimum-phase linear uncertain systems with quantized measurements, a new passification-based adaptive controller has been proposed. The novelty of the controller is in the switching procedure for quantizer zooming and particular parameters that are involved in the adaptation law. The switching instants and values of switched parameters can be calculated "a priori" using available information. It has been proved that if an estimate of the initial conditions is known and the quantization error bound that is calculated from this estimate is small enough then the state of a hyper-minimum-phase system can be made arbitrarily small. Moreover, when applying to polytopic-type uncertain systems, the proposed adaptive controller results in a smaller controller gain, what gives it an advantage over the static feedback control. This was demonstrated by the example of yaw angle control of an aircraft.

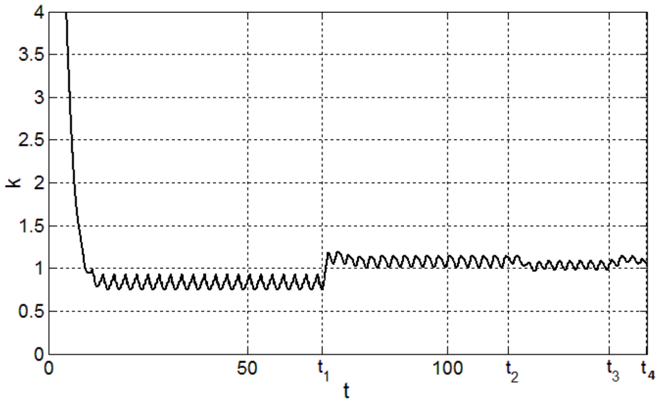

Fig. 2. Evolution of $k(t)$.

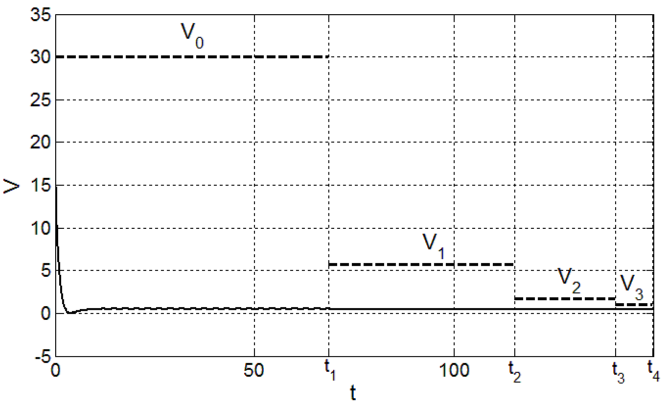

Fig. 3. Evolution of $V(x(t), k(t))$ given by (14).

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